[tex9] **Work extracted from finite heat reservoir in infinite environment**

A (finite) heat reservoir with heat capacity $C =$ const is initially at temperature T_H and the (infinite) environment at the lower temperature T_0.

Now the reservoir is connected to the environment by a heat engine, which absorbs an infinitesimal amount of heat δQ per cycle, converts part of it into work δW, and dumps the rest into the environment. During each cycle the temperature of the reservoir decreases infinitesimally: $\delta Q = -CdT$.

Determine the maximum amount of work ΔW that can be extracted from the reservoir before its temperature has dropped to that of the environment.

The fraction of the excess internal energy $U_{ex} = C(T_H - T_0)$ that can be converted into work is characterized by the quantity $\Delta W/U_{ex}$. Plot this quantity versus the reduced temperature $(T_H - T_0)/T_0$ for $T_0 < T_H < 3T_0$. Discuss the properties of this function in the limit $T_H \to T_0$.