[tex92] **Relativistic ideal gas (entropy and internal energy)**

(a) Derive from the result for the canonical partition function $Z_N$ of the relativistic classical ideal gas as calculated in [tex91] the Helmholtz free energy $A(T,V,N)$, the equation of state $p(T,V,N)$, the chemical potential $\mu(T,V,N)$, and the entropy $S(T,V,N)$.

(b) Use the recursion relation, $K_n'(u) = -K_{n-1}(u) - (n/u)K_n(u)$, for the modified Bessel functions to derive the following expression for the internal energy:

$$U(T,N) = Nk_BTu\left[\frac{K_1(u)}{K_2(u)} + \frac{3}{u} - 1\right],$$

$$u \equiv \beta mc^2, \quad \beta = \frac{1}{k_BT}.$$