[tex95] **Density fluctuations in the grandcanonical ensemble**

Consider a system of indistinguishable particles in the grandcanonical ensemble. Derive the following two expressions for the fluctuations in the number of particles N for an open system of volume V in equilibrium with heat and particle reservoirs at temperature T and chemical potential μ, respectively:

$$\langle N^2 \rangle - \langle N \rangle^2 = z\frac{\partial}{\partial z} z\frac{\partial}{\partial z} \ln Z = k_B T V \frac{\partial^2 p}{\partial \mu^2},$$

where $z = \exp(\mu/k_B T)$ is the fugacity,
$p(T,V,\mu) = -(\partial\Omega/\partial V)_{T\mu} = -\Omega/V$ is the pressure, and $\Omega(T,V,\mu) = -k_B T \ln Z$ is the grand potential.