BE gas in $D$ dimensions IV: heat capacity at high temperature

The internal energy of the ideal Bose-Einstein gas in $D$ dimensions and at $T \geq T_c$ is given by the following expression:

$$U = N k_B T \frac{D}{2} \frac{g_{D/2+1}(z)}{g_{D/2}(z)}.$$ 

Use this result to derive the following expression for the heat capacity $C_V = \langle \partial U / \partial T \rangle_{V,N}$:

$$C_V = \frac{N k_B}{V} \left( \frac{D}{2} + \frac{D^2}{4} \right) \frac{g_{D/2+1}(z)}{g_{D/2}(z)} - \frac{D^2}{4} \frac{g_{D/2}(z)}{g_{D/2-1}(z)}.$$ 

Use the derivative $\partial / \partial T$ of the result $g_{D/2}(z) = \mathcal{N} \lambda_T^D / V$ with $V = L^D$ to calculate any occurrence of $\langle \partial z / \partial T \rangle_{V,N}$ in the derivation. Use also the recursion relation $g_n' = g_{n-1}(z)$.

Solution: