[tex97] **Heat capacity of ideal Bose–Einstein gas**

The internal energy of the ideal Bose-Einstein gas is given by the following expressions:

$$U = \frac{3}{2}\mathcal{N}k_BT\frac{g_{5/2}(z)}{g_{3/2}(z)}, \quad (T > T_c);$$

$$U = \frac{3}{2}k_BT\frac{V}{\lambda_T^3}\zeta(5/2), \quad (T < T_c).$$

(a) Use these results to derive the following expressions for the heat capacity $C_V = (\partial U/\partial T)_{V\mathcal{N}}$:

$$\frac{C_V}{\mathcal{N}k_B} = \frac{15}{4}\frac{g_{5/2}(z)}{g_{3/2}(z)} - \frac{9}{4}\frac{g_{3/2}(z)}{g_{1/2}(z)}, \quad (T > T_c);$$

$$\frac{C_V}{\mathcal{N}k_B} = \frac{15}{4}\frac{V}{\mathcal{N}}\frac{\zeta(5/2)}{\lambda_T^3}, \quad (T < T_c).$$

To accomplish this task, use the recursion relation $zg_n'(z) = g_{n-1}(z)$ for the Bose-Einstein functions and use the derivative $\partial/\partial T$ of the result $g_{3/2}(z) = \mathcal{N}\lambda_T^3/V$ to calculate any occurrence of $(\partial z/\partial T)_{V\mathcal{N}}$ in the derivation.

(b) Show that the two results for C_V assume the same value at $T = T_c$ and determine that value.