[tex98] **Ultrarelativistic Bose–Einstein gas**

Consider a Bose-Einstein gas with ultrarelativistic one-particle energy $\epsilon_k = c\hbar k = cp$ in the grandcanonical ensemble at temperature T and chemical potential $\mu = 0$.

(a) Show that the one-particle density of states is $D(\epsilon) = (4\pi V/h^3c^3)\epsilon^2$.

(b) Calculate the pressure $p(T)$, the internal energy $U(T,V)$, and the average number of particles in excited states $\mathcal{N}_\epsilon(T,V)$.

(c) Infer from these results the equation of state $pV/\mathcal{N}_\epsilon = \pi^4/90\zeta(3) \simeq 0.900$.

(d) Show that the heat capacity is
$C_V/k_B = [48\pi^5/45h^3c^3]V(k_BT)^3$.