

[mex110] **Balancing a heavy object on a light rod**

The equations of motion

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \frac{c_1 x_1}{L} \cos x_1 + \frac{c_2 x_2}{L} \cos x_1 + \frac{g}{L} \sin x_1$$

represent a point mass m being balanced on a rod of length L and negligible mass through lateral movement of the pivot. Here the variable x_1 represents the angle ϕ from the upright equilibrium position and the variable x_2 the associated angular velocity $\dot{\phi}$ as explained in [mIn33].

(a) Analyze the nature of the fixed point at $(x_1, x_2) = (0, 0)$ for the case with zero feedback ($c_1 = c_2 = 0$).

(b) Determine the conditions for the control parameters c_1, c_2 under which the the fixed point at $(x_1, x_2) = (0, 0)$ is an attractor, i.e. for which it is asymptotically stable.