

[pex3] Electric force between charged plates immersed in electrolyte

Consider two conducting plates with uniform charge densities σ_A and σ_B positioned as shown.

(a) Show that the electric potential as calculated in [pex4] has a minimum at position x_0 , where

$$\tanh(\kappa x_0) = \frac{\psi_A \cosh(\kappa h) - \psi_B}{\psi_A \sinh(\kappa h)}. \quad (1)$$

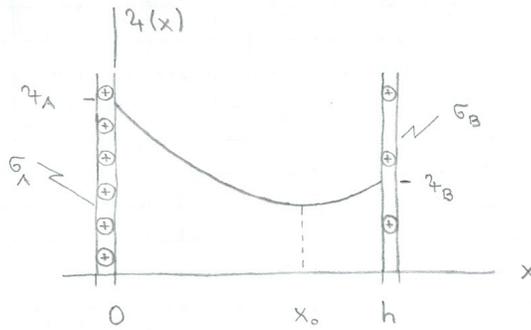
(b) Calculate the interaction force (per unit area) between the plates, using the relation,

$$f_{\text{int}} = \frac{1}{2} \epsilon \kappa^2 [\psi(x_0)]^2, \quad \left. \frac{d\psi}{dx} \right|_{x_0} = 0, \quad (2)$$

justified in [pln78], arriving at the result

$$f_{\text{int}} = \frac{1}{2} \epsilon \kappa^2 \frac{2\psi_A \psi_B \cosh(\kappa h) - \psi_A^2 - \psi_B^2}{\sinh^2(\kappa h)}. \quad (3)$$

with the dependence of ψ_A, ψ_B on σ_A, σ_B as determined in [pex4] and stated in [pln78].



[adapted from Doi 2013]

Solution: