[pex32] Ideal polymer chain: flexibility from rigid constraints

Here we connect the results for the mean-square end-to-end distance,

$$\langle R^2 \rangle = 2 l_\mathrm{p} \left[L - l_\mathrm{p} \big(1 - e^{-L/l_\mathrm{p}} \big) \right], \tag{1}$$

for a flexible polymer chain with contour length L and persistence length l_p from [pex29] and,

$$\langle R^2 \rangle = N a^2 \left[\frac{1+\cos\gamma}{1-\cos\gamma} - N^{-1} \frac{2\cos\gamma}{1-\cos\gamma} \frac{1-(\cos\gamma)^N}{1-\cos\gamma} \right], \tag{2}$$

for a polymer chain with $N = L/a$ links of size a and fixed valence angle γ from [pex30]. The persistence length l_p for this model is calculated in [pex31]. Show that in the combined limit $\gamma \ll 1$ and $l_\mathrm{p} \gg a$ the result (2) for a chain with rigidly constrained valence angle turns into the result (1) for a chain whose flexibility can be described by a bending stiffness as in [pex28].

[adapted from Grosberg and Khokhlov 1994]

Solution: