[pex35] Polymer creep compliance: linear response

The basic model for creep compliance expresses the time-dependent strain $e(t)$ that results from a stress σ_0 turned on abruptly and held constant, $e(t) = J(t)\sigma_0$. For a viscoelastic material, the creep compliance $J(t)$ is a monotonically increasing function that rises very steeply from zero and then approaches a more moderate, linear increase at large t. For situations with time-dependent stress $\sigma(t)$, this linear response generalizes into the relation (Boltzmann superposition principle)

$$e(t) = \int_{-\infty}^{t} d\tau\, J(t-\tau)\frac{d\sigma(\tau)}{d\tau}.$$

Here we consider the following two-parameter creep compliance representing the viscoelastic behavior of some hypothetical polymer melt:

$$J(t) = bt + c\sqrt{t}, \quad b, c > 0,$$

(a) Calculate the time-dependent strain, $e(t)$, in (linear) response to a stress that increases from zero at constant rate: $\sigma(t) = at$. Plot both $J(t)$ and $e(t)$ versus t for $a = 1, c = 1$, and $b = 0.1, 0.5, 1$. Interpret your results.
(b) Calculate the time-dependent strain, $e(t)$, in (linear) response to a harmonically oscillating stress: $\sigma(t) = a\sin(\omega t)$. Write the result in the form $e(t) = e_b(t)+e_c(t)$, where each term represents the linear response to one term of $J(t)$. Plot $e(t), e_b(t), e_c(t)$ versus t for $a = 1, c = 1, b = 0.2$, and $\omega = 0.5, 1, 2$. Produce three graphs for different values of ω, each with three curves. Use the range of t such that $0 < \omega t < 10$ in each graph. Interpret your results.
Note that the term "linear" is used or implied to describe three different aspects: (i) the linear term in $J(t)$, (ii) the linear relation between strain $e(t)$ and stress rate $d\sigma/d\tau$, (iii) the linear stress increase in $\sigma(t) = at$.

Solution: