[pex46] Chemical potential in two-component system

Consider a two-component fluid system with Gibbs free energy,

$$G = V\big[p + f(T,\phi)\big],$$

where $V = N_p v_p + N_s v_s$ is the volume and p the pressure. The numbers of solute and solvent particles are N_p and N_s, respectively. Their specific volumes are v_p and v_s, respectively. The free-energy density $f(T,\phi)$ is an unspecified function of temperature T and volume fraction $\phi = N_p v_p/V$ of solute particles.
(a) Derive, via standard thermodynamic relations $\mu_p = (\partial G/\partial N_p)_{T,p,N_s}$ and $\mu_s = (\partial G/\partial N_s)_{T,p,N_p}$, the following general expressions for the chemical potentials of the solute and solvent particles:

$$\mu_p(T,p,\phi) = v_p\big[p + f(T,\phi) + (1-\phi)f'(T,\phi)\big], \quad \mu_s(T,p,\phi) = v_s\big[p + f(T,\phi) - \phi f'(T,\phi)\big],$$

where we use the convention $f' \doteq \partial f/\partial\phi$.
(b) If the profile of $f(T,\phi)$ permits the coexistence of two phases with volume fractions $\phi_a < \phi_b$ then the common-tangent conditions must be satisfied: $f'(T,\phi_a) = f'(T,\phi_b)$ and $f(T,\phi_a) + f'(T,\phi_a)[\phi_b - \phi_a] = f(T,\phi_b)$. Show that it follows that the chemical potentials must be the same in both phases: $\mu_p(T,\phi_a) = \mu_p(T,\phi_b)$ for the solute and $\mu_s(T,\phi_a) = \mu_s(T,\phi_b)$ for the solvent.

Solution: