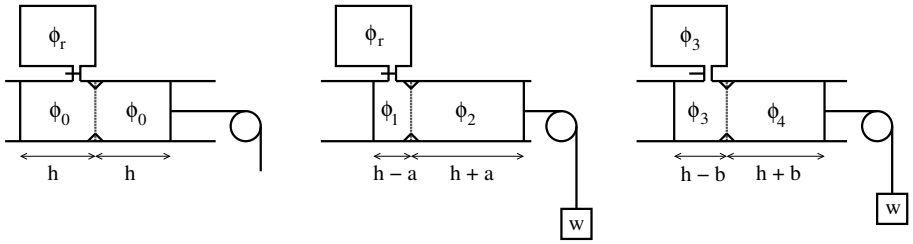
[pex49] Osmotic weight lifting

Consider a cylinder of cross-sectional area A. The space between two pistons, a distance $2h$ apart, is divided into two compartments by an immobile semi-permeable membrane. In the initial state both compartments have equal size and are filled with a solution of equal solute volume fraction $\phi_0 = 0.4$. The pistons are stabilized by ambient air pressure. A reservoir of volume equal to either initial compartment contains a more highly concentrated solution ($\phi_r = 0.9$).
When a force w is applied to one piston as shown, some solvent is forced through the membrane. This changes the solute concentrations in the two compartments until the difference in osmotic pressure, $\Delta\pi$, balances the applied force.
When the valve is opened, the solutions in the reservoir and in the compartment on the left are allowed to mix. This changes $\Delta\pi$ and lifts the weight w to a new balance of forces.
Use the expression $\pi(\phi) = p_0[-\ln(1-\phi) - \chi\phi^2]$ derived in [pex48] for the osmotic pressure and set $\chi = 1$, indicating strong solubility.
(a) Plot $\Delta\pi/p_0$ as a function of the relative displacement x/h (i) with the valve still closed and (ii) with the valve open. In both cases x is the diplacement of both pistons from their initial position.
(b) Assuming that the weight is $w = p_0 A$ calculate the equilibrium displacement $x = a$ before the valve has been opened and $x = b$ after the valve has been opened.



Solution: