

[pex5] Maier-Saupe theory III: first-order phase transition

This is the continuation of [pex44], where we have determined an explicit, parametric expression for the scaled free energy in terms of order parameter and entropy:

$$\Delta\hat{F}(b, \hat{u}) = -\frac{1}{2}\hat{u}[\mathcal{N}(b)]^2 - \Delta\hat{S}(b),$$

where b is the parameter and $\hat{u} = 1/\hat{T}$ is a scaled coupling constant.

The location of the free-energy minimum determines a function $\bar{\mathcal{N}}(\hat{T})$, which we examine here.

- (a) Locate the transition temperature $\hat{T}_c \doteq 1/\hat{u}_c$, as the inverse coupling strength for which the location of that minimum in $\Delta\hat{F}(b, \hat{u})$ switches between $\mathcal{N} = 0$ to a nonzero value of \mathcal{N} .
- (b) Design a way to produce an accurate graphical representation of the function $\bar{\mathcal{N}}(\hat{T})$.

[adapted from Jones 2002]

Solution: