

[pex9] Electric potential near thin layer of bound charge

Consider a thin layer of bound charge,

$$n_b(x) = n_l \delta(x), \quad (1)$$

with (positive) charge per unit area e_l immersed in a monovalent, ionic solvent with asymptotic densities $n_{\pm}^{(0)} = n_0$, consistent with charge neutrality.

(a) Show that the profile of the electric potential is of the exponential form,

$$\psi(x) = \frac{n_l e_l}{2\kappa\epsilon} e^{-\kappa|x|}, \quad \kappa^{-1} = \sqrt{\frac{\epsilon k_B T}{2n_0 e_0^2}}, \quad (2)$$

where κ^{-1} is the Debye screening length (see also [pex25]). The most expedient way to carry out this task is by confirmation that the function (2) is a solution of the linearized Poisson-Boltzmann equation from [pln68] with $n_b(x)$ from (1).

(b) Derive expressions for the densities $n_{\pm}(x)$ of mobile ions and sketch their dependence on x .

[adapted from Doi 2013]

Solution: