Linear response [nln26]

Radiation field $b(t)$ perturbs equilibrium state of the system $\mathcal{H}_0$ via coupling to dynamical variable B.

System response to perturbation measured as expectation value of dynamical variable A.

Linear response to weak perturbations is predominant under most circumstances (away from criticality).

Response function $\tilde{\chi}_{AB}(t)$ (definition):

$$\langle A(t)\rangle - \langle A\rangle_0 = \int_{-\infty}^{\infty} dt' \tilde{\chi}_{AB}(t-t') b(t').$$

- Linearity: $\tilde{\chi}_{AB}(t)$ is independent of $b(t)$.
- Hermiticity: $\tilde{\chi}_{AB}(t)$ is a real function.
- Causality: $\tilde{\chi}_{AB}(t) = 0$ for $t < 0$.
- Smoothness: $|\tilde{\chi}_{AB}(t)| < \infty$.
- Analyticity: $\tilde{\chi}_{AB}(t) \to 0$ for $t \to \infty$.

Generalized susceptibility (via Fourier transform):

$$\chi_{AB}(\zeta) = \int_{-\infty}^{+\infty} dt\, e^{\imath\zeta t} \tilde{\chi}_{AB}(t) \qquad \text{(analytic for } \Im\{\zeta\} > 0\text{)}.$$

Complex function of real frequency:

$$\chi_{AB}(\omega) = \lim_{\epsilon\to 0} \chi_{AB}(\omega + \imath\epsilon) = \chi'_{AB}(\omega) + \imath\chi''_{AB}(\omega).$$

Linear response in frequency domain means no mixing of frequencies:

$$\alpha(\omega) = \chi_{AB}(\omega)\beta(\omega),$$

where

$$\tilde{\chi}_{AB}(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-\imath\omega t} \chi_{AB}(\omega), \quad b(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-\imath\omega t} \beta(\omega),$$

$$\langle A(t)\rangle - \langle A\rangle_0 = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-\imath\omega t} \alpha(\omega).$$