Predator-Prey System [nsl3]

A population of foxes (predator F) feeds on a population of hares (prey H). The birth rate of foxes is proportional to the fox population and to the amount of food available. Foxes die naturally, i.e. at a rate proportional to the fox population. Hares die primarily through encounters with foxes and are born at a rate proportional to the hare population.

Deterministic time evolution: Lotka-Volterra model.

$$\frac{dH}{dt} = aH - bHF, \quad \frac{dF}{dt} = bHF - dF.$$

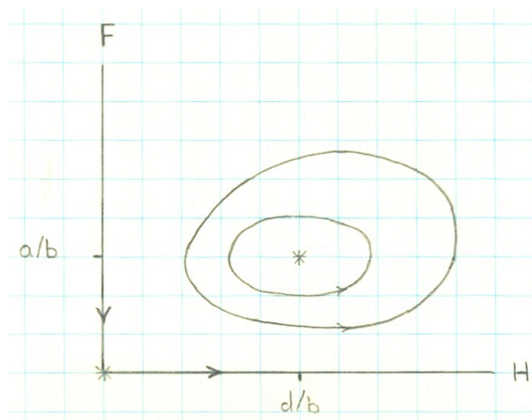
Anharmonic oscillation.

Hyperbolic fixed point at $(0, 0)$.

Elliptic fixed point at $(d/b, a/b)$.

Fluctuations excluded.

Environmental effects
in initial conditions only.



Stochastic time evolution: master equation.

$$\frac{\partial}{\partial t}P(H,F,t) = \sum_{H'F'}\Big[W(H|H';F|F')P(H',F',t) - W(H'|H;F'|F)P(H,F,t)\Big],$$

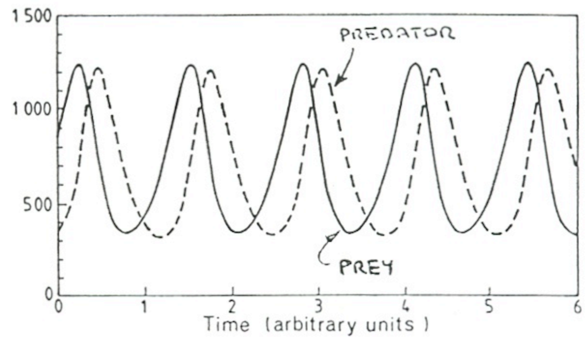
Non-vanishing transition rates:

- $W(H+1|H;F|F) = aH$ (prey is born)
- $W(H-1|H;F+1|F) = bHF$ (predator thrives on prey)
- $W(H|H;F-1|F) = dF$ (predator dies)

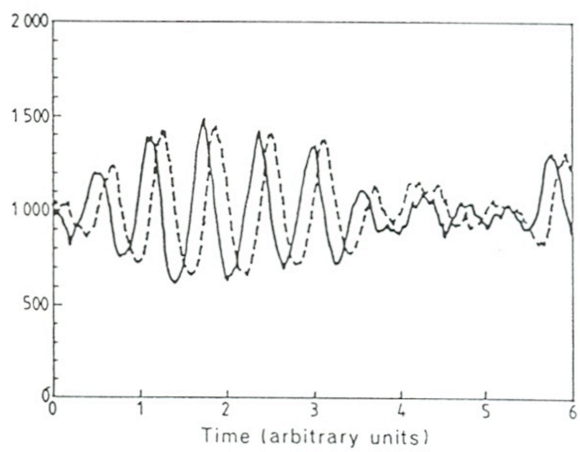
Fluctuations now included.

Environmental effects in intial conditions and in contingencies of stochastic time evolution.

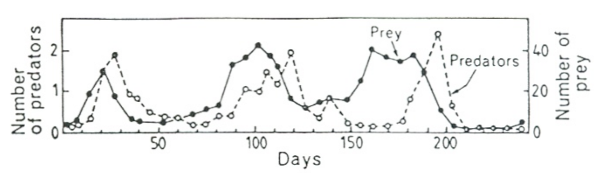
- Time evolution of deterministic system: Lotka-Volterra model



- Computer simulation of stochastic system: master equation



- Observation of real system: two species of mites



[images from Gardiner 1985]