[nex1] Subtlety of statistical independence

(a) Given that events A, B are statistically independent, $P(AB) = P(A)P(B)$, show that the event pairs $\bar{A}, B$ and $\bar{A}, \bar{B}$ are also statistically independent:

$$P(\bar{A}B) = P(\bar{A})P(B), \quad P(\bar{A}\bar{B}) = P(\bar{A})P(\bar{B}).$$

(b) Consider three events A, B, C occurring with probabilities $P(A), P(B), P(C)$, respectively, and satisfying the relations,

$$P(AB) = P(A)P(B),\ P(BC) = P(B)P(C),\ P(CA) = P(C)P(A).$$

Show, by counterexample, that these relations are compatible with the relation

$$P(ABC) \neq P(A)P(B)P(C),$$

in which case the three events are not statistically independent.

Hint: Create (by demonstration in Venn diagram) three nonidentical events A, B, C with intersections $AB = AC = BC = ABC$.

Solution: