

[nex101] Fokker-Planck equation with constant coefficients

(a) Convert the Fokker-Planck equation with constant coefficients of drift and diffusion,

$$\frac{\partial}{\partial t} f(x, t|x_0) = -A \frac{\partial}{\partial x} f(x, t|x_0) + \frac{1}{2} B \frac{\partial^2}{\partial x^2} f(x, t|x_0),$$

into an ordinary differential equation for the characteristic function,

$$\Phi(k, t) \doteq \int_{-\infty}^{+\infty} dx e^{ikx} f(x, t|x_0).$$

(b) Solve this differential equation (by elementary means) and infer $P(x, t|x_0)$ via inverse Fourier transform. Use the initial condition $P(x, 0|x_0) = \delta(x - x_0)$.

(c) Identify the mean $\langle\langle x \rangle\rangle$ and the variance $\langle\langle x^2 \rangle\rangle$ in the solution $P(x, t|x_0)$.

Solution: