[nex103] House of the mouse: some one-way doors
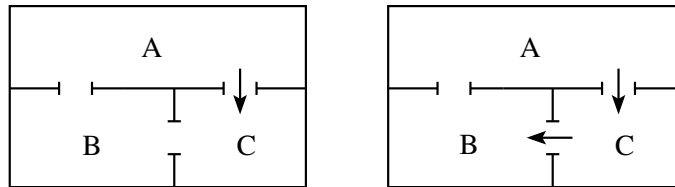
A trained mouse lives in a house with floor plan as shown in two versions. The house has three rooms and three doors. One or two doors are open one way only. A bell rings at regular time intervals, prompting the mouse to go to an adjacent room through any open door with equal probability.

(a) Construct the transition matrix $\mathbf{W}$ for both floor plans.

(b) The one-way doors are incompatible with the detailed-balance condition. Show that the transition matrix is regular, nevertheless, in both cases. For what minimum exponent s does $\mathbf{W}^s$ have no zero elements in each case?

(c) Regularity of $\mathbf{W}$ guarantees that the probability distribution for the location of the mouse is unique after the bell has rung a great many times. Calculate that stationary distribution for both cases by solving the left-eigenvector problem of matrix $\mathbf{W}$.

Solution: