

**[nex106] Life expectancy of the young and the old**

The distribution of lifetimes in some population is  $f(t) = (4t/T^2)e^{-2t/T}$ .

- (a) Show that  $f(t)$  is properly normalized and that the parameter  $T$  is the average lifetime of individuals.  
(b) Calculate the conditional probability distribution  $P_c(t|\tau)$

$$P_c(t|\tau) = \frac{f(t)}{C(\tau)}\theta(t - \tau), \quad C(\tau) \doteq \int_{\tau}^{\infty} dt f(t),$$

for the remaining lifetime of individuals of age  $\tau$ .

- (c) Calculate the *life expectancy*  $T_{\tau}$ , defined as the average remaining lifetime for an individual of age  $\tau$ , as a function of  $T$  and  $\tau$ .  
(d) What is the life-expectancy ratio  $T_{\infty}/T_0$  of the very old and the very young.

**Solution:**