[nex108] Isomeric transition III: total rate of transitions

A molecule exist exists in two isomeric arrangements of atoms, X and Y. In the presence of a catalyst, transitions happen at rates K_1 and K_2:

$$X \underset{K_2}{\overset{K_1}{\rightleftarrows}} Y.$$

The approach to equilibrium is governed by a master equation (12) with transition rates,

$$W(m|n) = K_1 n\delta_{m,n-1} + K_2(N-n)\delta_{m,n+1},$$

where the number of isomers are $n_X = n$ and $n_Y = N - n$ with constant N. Processes of given overall time scale depend on a single parameter, controlled by the density of the catalyst. We set

$$K_1 = \gamma, \quad K_2 = 1-\gamma \quad : \ 0 < \gamma < 1.$$

The total rate of isomeric transitions is defined as follows:

$$R(t) \doteq \sum_{nm} W(n|m)P(m,t).$$

(a) Express $R(t)$ in terms of time-dependent mean $\langle n(t)\rangle$ determined in [nex107].
(b) Relate the rate $R(t)$ in the long-time asymptotic limit $t \to \infty$ to the stationary variance $\langle\langle n^2\rangle\rangle_S$ determined in [nex46].
(c) Show that $R(t)$ is time-independent for arbitrary initial conditions if $\gamma = \frac{1}{2}$.

Solution: