[nex110] leaky gas tank III: evolution of mean and variance

A gas tank of volume V has a small leak and exchanges gas atoms with a large environment at constant particle density ρ. The master equation for the PMF $P(n,t)$ of atoms in the tank is specified by transition rates of the form

$$W(m|n) = \rho\,\delta_{m,n+1} + \frac{n}{V}\,\delta_{m,n-1}.$$

The generating function for initial condition $P(n,0) = n_0$ calculated in [nex48] is

$$G(z,t) = e^{V\rho(z-1)\left[1-e^{-t/V}\right]}\left[e^{-t/V}(z-1)+1\right]^{n_0}.$$

(a) Calculate $\langle n(t)\rangle_f$ and $\langle n^2(t)\rangle_f$ directly from the derivatives of $G(z,t)$ and infer time-dependent mean and variance from these factorial moments.
(b) Calculate the first two jump moments,

$$\alpha_l(m) = \sum_n (n-m)^l W(n|m) \quad : \; l = 1,2,$$

from the transition rates and solve the associated equations of motion,

$$\frac{d}{dt}\langle n\rangle = \langle\alpha_1(n)\rangle, \quad \frac{d}{dt}\langle n^2\rangle = \langle\alpha_2(n)\rangle + 2\langle n\alpha_1(n)\rangle.$$

Confirm that the results agree with those of part (a).

Solution: