[nex114] Modified linear birth rate II: evolution of mean and variance

Consider the master equation of a linear birth-death process with a modified birth rate,

$$W(m|n) = \lambda(n+1)\delta_{m,n+1} + \mu n \delta_{m,n-1}.$$

(a) Determine the first two jump moments,

$$\alpha_l(n) = \sum_m (m-n)^l W(m|n) \ : \ l = 1, 2.$$

(b) Calculate the time evolution of the mean $\langle n \rangle$ and the variance $\langle\langle n^2 \rangle\rangle$ with vanishing initial values by solving the equations of motion for the expectation values,

$$\frac{d}{dt}\langle n \rangle = \langle \alpha_1(n) \rangle, \qquad \frac{d}{dt}\langle n^2 \rangle = \langle \alpha_2(n) \rangle + 2\langle n \alpha_1(n) \rangle.$$

(c) Plot the functions $\langle n(t) \rangle$ and $\langle\langle n^2(t) \rangle\rangle$ for one case each with $\lambda > \mu$, $\lambda = \mu$, and $\lambda < \mu$. Describe the long-time asymptotics for each curve of each case.

Solution: