[nex117] Thermal diffusivity

A solid wall of very large thickness and lateral extension (assumed to occupy all space at $z > 0$) is brought into contact with a heat source at its surface ($z = 0$). The wall is initially in thermal equilibrium at temperature T_0. The heat source is kept at the higher temperature T_1. The contact is established at time $t = 0$. Show that the temperature profile inside the wall depends on time as follows:

$$T(z) = T_0 + (T_1 - T_0)\mathrm{erfc}\left(\frac{z}{2\sqrt{D_T t}}\right),$$

where $D_T = \lambda/c_V$ is the thermal diffusivity, λ the thermal conductvity, and c_V the specific heat. Then plot $T(z)/T_0$ versus z for $0 \leq z \leq 5$, $T_1/T_0 = 3$, and $D_T t = 0.2, 1, 5$. Describe the meaning of the three curves in relation to each other. The complementary error function is defined as follows:

$$\mathrm{erfc}(x) \doteq \frac{2}{\sqrt{\pi}} \int_x^\infty du\, e^{-u^2}.$$

Solution: