[nex15] **Binomial distribution and Poisson distribution.**

Consider the binomial distribution for two event $A, B$ that occur with probabilities $P(A) \equiv p$, $P(B) = 1 - p \equiv q$, respectively:

$$P_N(n) = \frac{N!}{n!(N-n)!}\, p^n q^{N-n},$$

where $N$ is the number of (independent) experiments performed, and $n$ is the stochastic variable that counts the number of realizations of event $A$.

(a) Find the mean value $\langle n \rangle$ and the variance $\langle\langle n^2 \rangle\rangle$ of the stochastic variable $n$.

(b) Show that for $N \to \infty$, $p \to 0$ with $Np \to a > 0$, the binomial distribution turns into the Poisson distribution

$$P_\infty(n) = \frac{a^n}{n!}\, e^{-a}.$$