[nex35] **Random frequency oscillator.**

Consider a physical ensemble of classical harmonic oscillators with randomly distributed angular frequencies: $P_\Omega(\omega) = \frac{1}{2}\Theta(1-|\omega|)$. At time $t = 0$ all oscillators are excited in phase with unit amplitude: $Y(t) = \cos(\omega t)$.

(a) Find the average displacement $\langle Y(t)\rangle$ and its variance $\langle\langle Y^2(t)\rangle\rangle$ as functions of t. What are the long-time asympotic values of these two quantities?

(b) Find the autocorrelation function $\langle Y(t+\tau)Y(t)\rangle$ for arbitrary t, τ and its asymptotic τ-dependence for $t \to \infty$.

(c) Show that the probability distribution of Y for $m\pi \leq t < (m+1)\pi$ is

$$P(y,t) = \frac{m}{t\sqrt{1-y^2}}\Theta(1-|y|) + \frac{1}{t\sqrt{1-y^2}}\Theta(y_{max}-y)\Theta(y-y_{min}),$$

where $y_{max} = 1$, $y_{min} = \cos t$ if $m = 0, 2, 4, \ldots$ and $y_{max} = \cos t$, $y_{min} = -1$ if $m = 1, 3, 5, \ldots$. Find the asymptotic distribution $P(y, \infty)$.