[nex40] Random walk in Las Vegas: chance and necessity

A gambler with \$1 in his pocket starts playing a game against a casino with infinite monetary resources. In each round of the game, the gambler wins \$1 (with probability p) or loses \$1 (with probability $1 - p$). The game ends when the gambler is bankrupt.
(a) Express the probability P_C that the gambler goes bankrupt eventually as a function of p.
(b) Plot P_C versus p for $0 < p < 1$.
(c) For what value of p is it a fair game in the sense that the gambler has a 50% chance of staying in the game forever?

Solution: