[nex44] **Linear birth−death process.**

Consider the master equation for the probability distribution $P(n,t)$ of the linear birth-death process. It is specified by the transition rates $W(m|n) = n\lambda\delta_{m,n+1} + n\mu\delta_{m,n-1}$, where λ and μ represent the (linear) birth and death rates, respectively, of individuals in some population. Calculate the time evolution of the mean value $\langle n\rangle$ and the variance $\langle\langle n^2\rangle\rangle$ for the initial condition $P(n,0) = \delta_{n,n_0}$ by solving the equations of motion for the expectation values $d\langle n\rangle/dt = \langle\alpha_1(n)\rangle$, $d\langle n^2\rangle/dt = \langle\alpha_2(n)\rangle + 2\langle n\alpha_1(n)\rangle$ with jump moments $\alpha_l(n) = \sum_m (m-n)^l W(m|n)$. Plot $\langle\langle n^2\rangle\rangle$ versus t for the three cases $\lambda > \mu, \lambda = \mu, \lambda < \mu$.