[nex47] **Ultracold neutrons in an ideal Steyerl bottle.**

Consider a container whose walls are perfect mirrors for ultracold neutrons. At time $t = 0$ the bottle is known to contain exactly n_0 neutrons (with decay rate K).

(a) Set up the master equation for the probability distribution $P(n,t)$, and derive the linear partial differential equation (PDE) for the associated generating function $G(z,t)$.

(b) Solve the PDE to obtain the function $G(z,t)$ and infer the distribution $P(n,t)$ from $G(z,t)$.

(c) Determine (via derivatives of the generating function) the average number $\langle n \rangle$ of remaining neutrons and the variance $\langle\langle n^2 \rangle\rangle$ thereof as functions of time.

(d) Derive the equations of motion for $\langle n \rangle$ and $\langle n(n-1) \rangle$ from the master equation and solve them to reproduce the solutions obtained in (e).