

[nex48] **Air escaping from a leaking container.**

Consider a box of volume  $V$ . At time  $t = 0$  it contains  $n_0$  molecules of air. The box has a small leak and exchanges molecules with the environment, which has a constant density  $\rho$  of air molecules.

(a) Set up the master equation for the probability distribution  $P(n, t)$  under the assumption that a molecule leaves the box with probability  $(n/V)dt$  and enters the box with probability  $\rho dt$ . Derive the linear partial differential equation (PDE) for the associated generating function  $G(z, t)$ .

(b) Solve the PDE to obtain the function  $G(z, t)$ .

(c) Determine (via derivatives of the generating function) the quantities  $\langle n \rangle$  and  $\langle\langle n^2 \rangle\rangle$  as functions of time and their asymptotic values for  $t \rightarrow \infty$ .

(d) Show that  $P(n, t)$  approaches the Poisson distribution for  $t \rightarrow \infty$ .