[nex5] **Bertrand's paradox analyzed**

Consider a circle of unit radius and draw *at random* a straight line intersecting it in a chord of length L

(a) by taking lines through an arbitrary fixed point on the circle with random orientation;

(b) by taking lines perpendicular to an arbitrary diameter of the circle with the point of intersection chosen randomly on the diameter;

(c) by choosing the midpoint of the chord at random in the area enclosed by the circle.

For each *random choice* determine the probability distribution $P(L)$ for the length of the chord and calculate the average length $\langle L \rangle$.