[nex50] Photon absorption and emission at thermal equilibrium

Consider a quantum harmonic oscillator in thermal equilibrium at temperature T. We know from [nex22] that the population of energy levels $E_n = n\hbar\omega_0, n = 0, 1, 2, \ldots$ is described by the geometric distribution,

$$P_S(n) = (1-\gamma)\gamma^n, \quad \gamma = e^{-\hbar\omega_0/k_BT}.$$

Show that this stationary PMF is ensues from the master equation of a birth-death process with transition rates,

$$W(m|n) = T_+(n)\delta_{m,n+1} + T_-(n)\delta_{m,n-1},$$

under the following conditions:

- The birth rate $T_+(n)$ represents photon absorption and the death rate $T_-(n)$ spontaneous and stimulated photon emission:

$$T_+(n) = (n+1)BN(\omega_0), \quad T_-(n) = n\big[A + BN(\omega_0)\big].$$

- The quantity $N(\omega_0)$ represents a bosonic density of photons at frequency ω_0:

$$N(\omega_0) = \frac{A/B}{e^{\hbar\omega_0/k_BT} - 1}.$$

Solution: