[nex51] **Stationary state for nonlinear birth–death process.**

Consider the master equation of the birth-death process with transition rates

$$W(m|n) = (n+1)\lambda\delta_{m,n+1} + \left[n\mu + \frac{\gamma}{N}n(n-1)\right]\delta_{m,n-1}.$$

It describes a population with a linear birth rate, $(n+1)\lambda$, and a linear death rate, $n\mu$. To account for the unhealthy environment under crowded circumstances ($n \simeq N$), a nonlinear death rate has been added to the process. Unlike the linear birth-death process, this process reaches a stationary distribution $P_S(n)$ in the case $\lambda > \mu$, i.e. when the linear birth rate exceeds the linear death rate.

Use the recurrence relation $P_S(n) = [T_+(n-1)/T_-(n)]P_S(n-1)$ to determine $P_S(n)$ for the following parameter values: $\lambda = 2, \mu = 1, \gamma = 1, N = 20$. Plot $P_S(n)$ versus n for $n = 0, 1, 2, \ldots, 50$. Find $\langle n\rangle$ and $\langle\langle n^2\rangle\rangle$ for this case.