[nex52] **Bistable chemical system.**

Consider the master equation of the birth-death process specified by transition rates of the form $W(m|n) = T_+(n)\delta_{m,n+1} + T_-(n)\delta_{m,n-1}$ with

$$T_+(n) = k_1 An(n-1) + k_3 A,\ T_-(n) = k_2 n(n-1)(n-2) + k_4 n.$$

This process describes two simultaneous chemical reactions $A + 2X \leftrightarrow 3X, A \leftrightarrow X$ that exhibit bistable states in a certain parameter range. The concentration of A is taken to be constant.

(a) Formulate the recurrence relation $P_S(n) = [T_+(n-1)/T_-(n)]P_S(n-1)$ for the equilibrium distribution $P_S(n)$ with use of the three parameters
$B = k_1 A/k_2, R = k_4/k_2, Q = k_3/k_1$.

(b) Show that that for $R/Q = 1$, these recurrence relations yield a Poisson distribution.

(c) Plot the solution of $T_+(n-1) = T_-(n)$ as a graph B versus n over the range $0 \leq n \leq 100$ for $Q = 100$ and the two values (i) $R = 500$ and (ii) $R = 1200$. Each point on such a graph indicates an extremum of $P_S(n)$. Note that for the case (ii), there exist three extrema for certain values of B.

(d) Plot $P_S(n)$ versus n over the range $0 \leq n \leq 100$ for $Q = 100$, $B = 70$ and the two values (i) $R = 500$ and (ii) $R = 1200$. Determine the position and height of the single peak in case (i) and of the two peaks in case (ii).