[nex62] **Fokker–Planck eq. for Brownian particle: eigenvalue problem.**

Consider the eigenvalue problem $\hat{\Omega}F(\mathbf{v}) = \lambda F(\mathbf{v})$ associated with the Fokker-Planck equation

$$\frac{\partial}{\partial t}\,P(\mathbf{v},t) = \hat{\Omega}P(\mathbf{v},t), \quad \hat{\Omega} = \xi\nabla_v \cdot \left(\mathbf{v} + \frac{k_B T}{m}\nabla_v\right)$$

for the (spatially uniform) velocity distribution $P(\mathbf{v},t) = F(\mathbf{v})e^{\lambda t}$ of Brownian particles. Show that if we set $F(\mathbf{v}) = \sqrt{P_S(\mathbf{v})}W(\mathbf{v})$, where
$P_S(\mathbf{v}) = (m/2\pi k_B T)^{3/2}\exp(-mv^2/2k_B T)$ is the stationary solution, then the eigenvalue equation satisfied by $W(\mathbf{v})$ is equivalent to the time-independent Schrödinger equation for the 3D harmonic oscillator.