

[nex64] Reactive and absorptive parts of linear response.

In the framework of linear response theory for $H = H_0 - a(t)A$, the rate of energy transfer between the system and the radiation field is

$$\frac{d}{dt}\langle H_0 \rangle = \int_{-\infty}^{\infty} dt' a(t)a(t') \frac{\partial}{\partial t} \tilde{\chi}_{AA}(t-t'), \quad (1)$$

where

$$\tilde{\chi}_{AA}(t-t') = \frac{i}{\hbar} \theta(t-t') \langle [A(t), A(t')] \rangle_0 \quad (2)$$

is the Kubo formula for the response function.

(a) Evaluate this expression for a monochromatic perturbation,

$$a(t) = \frac{1}{2} a_m (e^{i\omega_0 t} + e^{-i\omega_0 t}) \quad (3)$$

and express it in terms of the reactive part, $\chi'_{AA}(\omega)$, and the absorptive (dissipative) part, $\chi''_{AA}(\omega)$, of the generalized susceptibility $\chi_{AA}(\omega)$.

(b) Show that the time-averaged energy transfer depends only on the absorptive part of $\chi_{AA}(\omega)$:

$$\overline{\frac{d}{dt}\langle H_0 \rangle} = \frac{1}{2} a_m^2 \omega_0 \chi''_{AA}(\omega_0). \quad (4)$$

Solution: