[nex7] **Law of large numbers**

Let $X_1, \ldots, X_N$ be N statistically independent random variables described by the same probability distribution $P_X(x)$ with mean value $\langle X \rangle$ and standard deviation $\sigma_X = \sqrt{\langle X^2 \rangle - \langle X \rangle^2}$. These random variables might represent, for example, a series of measurements under the same (controllable) conditions. The law of large numbers states that the uncertainty (as measured by the standard deviation) of the stochastic variable $Y = (X_1 + \cdots + X_N)/N$ is

$$\sigma_Y = \frac{\sigma_X}{\sqrt{N}}.$$

Prove this result.