[nex73] **Structure function of harmonic oscillator III.**

Consider the quantum harmonic oscillator (for $\hbar = 1$),

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega_0^2 q^2 = \omega_0 \left(a^\dagger a + \frac{1}{2} \right),$$

where $q = (a^\dagger + a)/\sqrt{2m\omega_0},\ p = i\sqrt{m\omega_0/2}(a^\dagger - a)$ relate the position and momentum operators ($[q,p] = i$) to the boson creation and annihilation operators ($[a, a^\dagger] = 1$).

Solve the equation of motion

$$\zeta \langle\langle A; B \rangle\rangle_\zeta^\pm = \langle [A, B]_\pm \rangle + \langle\langle [A, H]; B \rangle\rangle_\zeta^\pm$$

for (a) the Green's function $\langle\langle q; q \rangle\rangle_\zeta^-$ and (b) the Green's function $\langle\langle q; q \rangle\rangle_\zeta^+$. Infer from each Green's function the structure function $S_{qq}(\omega)$ for the position variable at temperature T.