[nex78] **Transformation between uniform and Gaussian distributions**

Consider two independent random variables X_1, X_2 that are uniformly distributed on the intervals $0 \leq x_1, x_2 \leq 1$. Show that the transformed variables

$$Y_1 = \sqrt{-2\ln X_1}\cos 2\pi X_2, \quad Y_2 = \sqrt{-2\ln X_1}\sin 2\pi X_2$$

obey independent normal distributions:

$$P_{\mathrm{Y}}(y_1, y_2) = \frac{1}{\sqrt{2\pi}}\, e^{-y_1^2/2}\, \frac{1}{\sqrt{2\pi}}\, e^{-y_2^2/2}.$$