[nex80] **Generating exponential and Lorentzian random numbers**

Given is a sequence of uniformly distributed random numbers $x_1, x_2, \ldots$ with $0 < x_1 < 1$ as produced by a common random number generator.

(a) Find the transformation $Z = Z(X)$ which produces an exponentially distributed sequence of random numbers $z_1, z_2, \ldots$.

(b) Find the transformation $Y = Y(X)$ which produces a Lorentzian distributed sequence of random numbers $y_1, y_2, \ldots$.

Each of the two resulting distributions should be parametrized by a positive constant as follows:

$$P_Z(z) = \frac{1}{\zeta}\, e^{-z/\zeta}, \qquad P_Y(y) = \frac{1}{\pi}\frac{a}{y^2 + a^2}.$$