[nex83] **Eigenfunctions of linearized RG transformation for $\alpha = 2$**

Show that the functions

$$\phi_n(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2} \mathsf{H}_n(x),$$

$$\mathsf{H}_n(x) = (-1)^n e^{x^2/2} \frac{d^n}{dx^n} e^{-x^2/2}, \quad n = 0, 1, 2, \ldots$$

are eigenfunctions with eigenvalues $\lambda_n = 2^{1-n/2}$ of the linearized renormalization group transformation for index $\alpha = 2$:

$$D_2 [\phi_n(x)] \equiv 2 \cdot 2^{1/\alpha} \int_{-\infty}^{+\infty} dy \; \phi_0 \left(2^{1/\alpha} x - y \right) \phi_n(y) = \lambda_n \phi_n(x).$$