PHY-420, Fall '09

Thermal & Statistical Physics

Homework # 1. Due on Thursday, September 24

1. Check calorie content of a typical chocolate bar.

a) How much hot water should you drink to get an equivalent calorie intake?

b) Compare this calorie content with the minimal work necessary to climb a mile-high mountain. How high should one chocolate bar or its hot water equivalent take you?

c) Does the result seem strange? Why? What is wrong?

2. Barometric equation

a) Consider a horizontal slab of air whose thickness (height) is dz. If this slab is at rest, the pressure holding it up from below must balance *both* the pressure from above plus the weight of the slab. Use this to find dP/dz, the variation of of pressure with altitude, in terms of the air density ρ.

b) Use the gas law to write the density of air in terms of pressure, temperature, and the average mass m of molecules [assume that air consists of 78% - by volume - of N_2, 21% of O_2, and 1% of argon]. Combine the result with part *a*) and derive the *barometric equation*

$$\frac{dP}{dz} = -\frac{mg}{k_B T} P. \tag{1}$$

c) Assume that the temperature of the atmosphere does not depend on the altitude. Solve the barometric equation (1) and show that the pressure depends exponentially on the altitude:

$$P(z) = P_0 \exp\left(-mgz/k_B T\right). \tag{2}$$

d) Get a similar equation for the density.

e) Assume that the pressure at sea level is 1 atm. Find the pressure at 1430 m above sea level, 3090 m, and 8850 m (Mt. Everest). When does the pressure change become noticeable?

f) What is the most important *additional* factor that affects the pressure change with the altitude? If you take this factor into account, will the pressures from the part *e*) become higher or lower? Do not try to make the calculations here; just give a qualitative answer.

3. Virial expansion and the van der Waals equation.

a) The usual gas equation is not very accurate. A systematic way to account for non-ideal behavior is the so-called *virial expansion*

$$PV = n_m RT \left(1 + \frac{B(T)}{V/n_m} + \frac{C(T)}{(V/n_m)^2} + ...\right) \tag{3}$$

Compute the second term in the virial expansion $B(T)/(V/n_m)$ for nitrogen at 100, 400, and 600 K assuming that the *second virial coefficient* $B(100) = -160$ cm^3/mol, $B(400) = 9$ cm^3/mol, $B(600) = 21.3$ cm^3/mol [the *first virial coefficient* is equal to 1]. How accurate is the gas equation at these temperatures?

b) The virial coefficients reflect the presence of forces between the molecules [we assume that these forces do not exist in ideal gases]. Can you explain why does the second virial coefficient change its sign with temperature?

c) Any relation between P, V, and T, like the ideal gas law, is called the **equation of state**. A relatively accurate equation of state is the **van der Waals equation**:

$$\left(P + \frac{an_m^2}{V^2}\right)(V - n_m b) = n_m RT \tag{4}$$

where a and b are gas-specific constants [we will discuss this equation and its origins in more detail later on]. Calculate the second virial coefficient $B(T)$ for a van der Waals gas in terms of constants a, b [Hint: Apply the binomial expansion

$$(1+x)^p \approx 1 + px + \frac{1}{2}p(p-1)x^2 + ... \text{ at } |px| \ll 1 \tag{5}$$

to $[1 - n_m b/V]^{-1}$.

4. Isotope separation for uranium.

Uranium has two common isotopes with atomic masses of 238 and 235. One way of the isotope separation for nuclear fuel and nuclear weapons is to exploit the difference in the average thermal speeds of the molecules of the uranium hexafluoride gas, UF_6. Calculate the *rms* speed of these molecules for both isotopes and compare them.

(This difference is so small that the isotope separation is a multistage process requiring numerous centrifuges!)

5. Explain the difference between $\langle v^2 \rangle$ and $\langle v \rangle^2$.

6. An ideal gas undergoes the following three-step cyclic process (problem 1.33). **A**: the gas expands at constant pressure; **B**: the pressure increases at constant volume; **C**: the volume and pressure decrease proportionally to each other back to the initial point. Draw the $P-V$ diagram of this cyclic process.

For each of the steps **A**, **B**, and **C** determine whether each of the following is positive, negative, or zero:

a) the work done on the gas

b) the change in the energy content of the gas

c) the heat added to the gas

What is the sign of each of these three quantities for the whole cycle?

What could be a purpose of such a process?

7. In a **Diesel engine,** atmospheric air is quickly compressed to 1/20 of its original volume.

a) Estimate the temperature of air after such compression and

b) explain why a Diesel engine does not need spark plugs.

8. If you do some research you can find that the power output of an ideal wind turbine P is (in proper units)

$$P \sim \frac{1}{2}\rho A u^3 \tag{6}$$

where ρ is the air mass density, A is the area swept by the blades, u is the wind velocity.

a) Choose reasonable values for u and a blade length for an industrial-size turbine and estimate the power output of the turbine.

b) How many such turbines do we need to build to replace *all* US electric power production of approximately 4,000 billions of kW-hours per year? How much land will all these wind farms occupy?

c) Is this feasible? Will this affect the wind patterns?

d) Explain the origin of Eq.(6).

e) What do you think are the main deficiencies of (6) and of its straightforward use in part *a*)? How would result change if you adopt a more realistic approach?