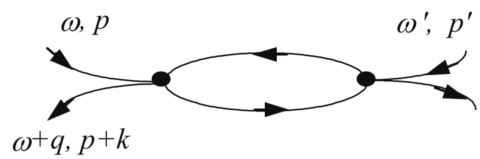

Figure 1

a) *b)*

Figure 2

Figure 1:

PHY-680, SPRING '08

CONDENSED MATTER PHYSICS II

Homework # 3. Due on Thursday, March 6.

1. Consider *ideal gas* of spin-$\frac{1}{2}$ fermions of mass m at $T = 0$ in magnetic field $\mathbf{B}$. Magnetic moment per particle is β, the overall density of gas is $n = N/V$.

a) Derive the *equations* that give you the Fermi momenta $p_{\uparrow,\downarrow}$, Fermi energies $\epsilon_{\uparrow,\downarrow}$, and densities $n_{\uparrow,\downarrow}$ of particles with up and down spins as a function of magnetic field. Choose proper dimensionless variables that best describe the problem.

b) Estimate numerically the magnetic field at which the the polarization saturates for a typical metal, liquid ^3He, and neutron gas with density 10^{19} particles/cm^3

c) Find the Green's function of this gas

2. Describe the 2-d order diagram for the vertex function in Figure 1 using the Green's functions from Problem 1. Add missing variables. Assuming that each vertex (dot) corresponds to the *s*-wave scattering with the scattering

amplitude a, give the analytical expression for the diagram *taking into account particle spin*. The scattering amplitude a does not depend on the particle momenta. Simplify the expression, but do not calculate the integrals at this point. The singularities of this vertex function are important for understanding superconductivity.

3. Consider the same ideal electron gas without magnetic field and disregard the spins. The so-called Hartree-Fock correction to the free electron Green's functions is, in fact, the lowest correction in perturbation theory and, as such, is given in diagrams *a*) and *b*) in Figure 2. The (forward scattering) part, given in 2*a*), is called the Hartree term and is exactly cancelled by the contribution of the uniform positive background in metals.

a) Why? How do you understand this statement?

This leaves us with the diagram *b*). Use for the interaction the Coulomb potential with the Fourier image $U(\mathbf{q}) = 4\pi e^2/q^2$.

b) Give the analytical expression for the Hartree-Fock **self-energy** Σ_{HF}.

c) Perform integration over the frequencies keeping in mind that in metal the electrons are located only inside the Fermi sphere.

d) Try to evaluate the remaining spatial integral.

4. *a*) Prove that for the Matsubara Green's functions $G^M(\tau) = \pm G^M(\tau + 1/T)$. Is it (+) or (-) ?

b) Prove that for the Matsubara Green's function $G^M(\tau)$ is periodic and find the period.