Theory of charge and impurity transfer in quantum crystals

A. F. Andreev and A. É. Meĭerovich

Institute of Physical Problems, USSR Academy of Sciences

(Submitted May 23, 1974)
Zh. Eksp. Teor. Fiz. 67, 1559–1567 (October 1974)

The mobility of ions and impurity atoms in quantum crystals of solid helium is studied. The quantum nature of the crystals makes it possible to elucidate the dependence of the mobility on temperature and external field without making any model assumptions for the ion structure. At high temperatures, the transfer results from elastic scattering of long-wave vacancions by impurities and ions. At low temperatures, the main mechanism of transfer is the proper tunneling of ions and impurities, which may be accompanied by scattering or spontaneous phonon emission. A peculiar dependence of the mobility on the direction and magnitude of the external field is found. The concentration dependence of the mobility is discussed.

The study of impurities in quantum crystals of solid helium is of considerable interest because, at sufficiently low temperature, any point defect becomes a quasiparticle that moves freely through the crystal[1,2]. In fact, we are only concerned with impurities of the other helium isotope and charges, since no other particles can be dissolved in helium in amounts sufficient to allow an experimental observation. A large amount of experimental work has been devoted to the study of transfer of charges[3-7] and of isotopic impurities[8-11]. In the latter case some peculiarities were observed which were brought about by the quantum nature of point defects.

The structure and mobility of negative ions in solid helium have been discussed theoretically in[12,13], where, in analogy to the familiar case of liquid, the negative ion is simulated by an electron placed in a cavity of macroscopic size. The cavity boundary, which lies on the surface of the crystal lattice, serves according to Shikin[13] as an effective source and sink of vacancies, and the visco-diffusive motion of the cavity is the mechanism of ion mobility. As to the positive ions, no satisfactory mechanism has been proposed so far to explain their motion. The analogy with the well-known case of liquid suggests that, owing to the striction forces, a higher-density region develops in the crystal around a positive ion (see[14]). The boundary of this region, however, does not lie on the surface of the crystal and, therefore, cannot serve as a source of vacancies. The visco-diffusive mechanism of motion considered by Mineev[14] is not possible in this case. Nevertheless, the experimental data[5-7] indicate that the values of mobility for ions of both signs are close to each other, and coincide in order of magnitude with the mobility of isotope impurities in the temperature region where the vacancion mechanism of motion is dominant. Furthermore, the radius of a negative ion, estimated in[12,13] does not exceed two or three atomic dimensions. Under these conditions the discreteness of the process of creation and destruction of vacancies by ions can hardly be neglected. The macroscopic boundary conditions on the ion surface, adopted in[13], correspond to neglect of precisely this factor.

An important feature disregarded in[13,14] is the large vacancion wavelength in solid helium. For not too high pressure and at all temperatures, it appreciably exceeds the interatomic separation and, therefore, the size of the ions. This results from the large (of order several degrees) value of the vacancion energy-band width, owing to which, for not too high temperature, the vacancions are located close to the bottom of the band.

It is precisely the last circumstance which enables us to explain the nature of the vacancion mobility of ions in its general form, without going into particulars of their microscopic structure. As will be shown below, the question of ion motion can be reduced to the well-known quantum-mechanical problem of inelastic scattering of slow particles. It then proves possible to evaluate the dependence of the mobility of both negative and positive ions on the temperature and on the applied electric field.

At sufficiently low temperatures the ion transport is due to tunneling transitions to the neighboring sites, for with decreasing temperature the vacancy concentration together with the vacancion mobility decreases exponentially.

At low temperature the ions turn into quasiparticles that move essentially freely through the crystal. This is connected with the fact that ions, as any object of atomic size in a quantum crystal, should be regarded as point defects, and the results of[1,2] can be applied to them. In this region, even for very small values of the electric field, the mobility must strongly depend on the field. This dependence is similar to that considered in[15] and is associated with the peculiar features of the motion of quasiparticles with a narrow energy band under the influence of a constant force. In particular, there exists a field range in which the drift velocity decreases as the field increases.

Everything said about the motion of ions can, in principle, be applied to any other point defect, in particular, to isotopic impurities. In this case, the part of electric field is played by, say, the magnetic field gradient (for He^3 impurities) or by nonuniform static deformation of the crystal which gives rise to a constant force acting on the point defects[15]. However, in contrast to the case of ions, whose concentration is in fact very small, the mobility of isotopic impurities may be influenced by their mutual scattering. This problem has been studied in[10,11], and the diffusion coefficient was found to be inversely proportional to the concentration of impurities. In the present work it is shown that, under certain conditions, the diffusion coefficient must be inversely proportional to concentration raised to the power $1/3$.

1. VACANCION MOBILITY

As stated above, at not too low temperatures the ion transport is due to the vacancion mechanism. A vacancy, while moving, can appear at a lattice site neighboring to an ion. The vacant site will thereupon be occupied by either a host atom or the ion, and the vacancy will then move away from the site of the ion delocalization. In a quantum-mechanical description of the interaction between delocalized vacancions and ions, these processes correspond to the scattering of delocalized particles by a localized object. The scattering will be either elastic or inelastic, depending on whether the ion remains localized at the initial lattice site or is shifted by an interatomic distance. In the course of inelastic scattering, the vacancion energy is changed by $e\mathbf{E}\cdot\mathbf{a}_n$, where e is the ion charge, $\mathbf{E}$ is the applied electric field, and $\mathbf{a}_n$ is the vector connecting the lattice site with its n-th nearest neighbor.

The average ion drift velocity u can be expressed in terms of the effective cross sections $\sigma_n(\mathbf{k})$ of such inelastic processes by the relation

$$\mathbf{u}=\sum_n \mathbf{a}_n \int \frac{d^3\mathbf{k}}{(2\pi)^3}\sigma_n(\mathbf{k})v(\mathbf{k})\{n(\varepsilon)-n(\varepsilon+e\mathbf{E}\mathbf{a}_n)\}, \qquad (1)$$

where k is the wave vector of the incident vacancion, $v(\mathbf{k})$ is its velocity, and $n(\epsilon)$ is the equilibrium vacancion distribution function over energies $\epsilon = \epsilon(\mathbf{k})$. Here and in what follows, the summation over n extends over those nearest neighbors $\mathbf{a}_n$ for which $e\mathbf{E}\cdot\mathbf{a}_n > 0$. The displacements of ions by $(-\mathbf{a}_n)$ are accounted for in (1) as inverse processes.

The expression (1) can be rewritten in the form:

$$\mathbf{u}=\sum_n \mathbf{a}_n \int \frac{d\varepsilon}{(2\pi\hbar)^3}\{n(\varepsilon)-n(\varepsilon+e\mathbf{E}\mathbf{a}_n)\}\int \sigma_n(\mathbf{k})\,dS, \qquad (2)$$

where the last integral is then taken over a surface of constant energy and is independent of n, owing to the symmetry of the crystal lattice.

It was stated in the Introduction that practically all vacancions are located close to the bottom of the energy band, where their spectrum is quadratic and the velocity of their motion is small. According to a well-known result of quantum mechanics, the inelastic-scattering cross section for slow particles is inversely proportional to their velocity. Hence

$$\int \sigma_n\,dS=4\pi\hbar\alpha(\varepsilon-\varepsilon_0)^{1/2},$$

where ϵ_0 is the energy corresponding to the bottom of the vacancion band, and the constant α is connected with the bandwidth Δ and the effective mass M by the relation $\Delta \sim \hbar/\sqrt{\Delta} \sim aM^{1/2}$ (a being the interatomic separation).

A Boltzmann distribution function $n(\epsilon)$ can be assumed, since ϵ_0 is of the order of the Debye temperature Θ and, hence, much exceeds the temperature T.

As a result we obtain

$$\mathbf{u}=\frac{\alpha}{4\hbar^2}\left(\frac{T}{\pi}\right)^{3/2}\exp\left(-\frac{\varepsilon_0}{T}\right)\sum_n \mathbf{a}_n\left[1-\exp\left(-\frac{e\mathbf{E}\mathbf{a}_n}{T}\right)\right]. \qquad (3)$$

In a weak electric field $eEa \ll T$ the drift velocity is proportional to the field

$$u_i=eB_{ik}E_k,$$

where the mobility tensor B_{ik}, by virtue of (3), is equal to

$$B_{ik}=\frac{\alpha}{4\pi\hbar^2}\left(\frac{T}{\pi}\right)^{1/2}e^{-\varepsilon_0/T}\sum_n a_{ni}a_{nk}. \qquad (4)$$

A rather peculiar situation arises in strong electric fields $eEa \gg T$. For practically every direction of $\mathbf{E}$ the velocity u reaches saturation and does not depend on $|\mathbf{E}|$:

$$\mathbf{u}=\frac{\alpha}{4\hbar^2}\left(\frac{T}{\pi}\right)^{3/2}e^{-\varepsilon_0/T}\sum_n \mathbf{a}_n. \qquad (5)$$

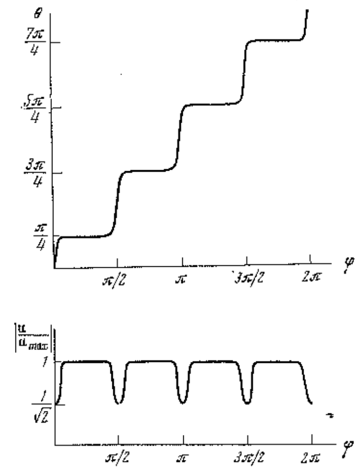
It is essential, however, that only those n for which $e\mathbf{E}\cdot\mathbf{a}_n > 0$ are to be summed over. Therefore, as the direction of the field varies, the direction of u changes practically in a step-like manner when $\mathbf{E}$ passes through a plane perpendicular to one of the vectors $\mathbf{a}_n$. The angular width of the transition region is of order $T/eEa \ll 1$. In such a transition region not only does the direction of u vary but its magnitude as well. As an example, the figure illustrates the dependence of both the magnitude and the direction of velocity on the orientation of $\mathbf{E}$ for a two-dimensional square lattice having two perpendicular fundamental translation vectors of equal length $\mathbf{a}_1$ and $\mathbf{a}_2$. In this case the maximum value of the drift velocity, u_{max}, according to (5), is given by

$$u_{max}=\frac{\sqrt{2}\,\alpha}{4\hbar^2}\left(\frac{T}{\pi}\right)^{3/2}e^{-\varepsilon_0/T}|\mathbf{a}_1|.$$

It has been assumed above that $|e\mathbf{E}\cdot\mathbf{a}_n| < \Delta$. For those $\mathbf{a}_n$ for which $e\mathbf{E}\cdot\mathbf{a}_n \to \Delta$ the expressions (3) and (5) should be refined. A vacancy can participate in transport processes only if $\epsilon(\mathbf{k}) - \epsilon_0 \leq \Delta - |e\mathbf{E}\cdot\mathbf{a}_n|$. A simple calculation shows that those terms in (3) and (5) which correspond to vectors $\mathbf{a}_n$ such that $\Delta - e\mathbf{E}\cdot\mathbf{a}_n \ll T$ must be multiplied by a small factor

$$\frac{4}{3\sqrt{\pi}}\left[\frac{\Delta-e\mathbf{E}\mathbf{a}_n}{T}\right]^{3/2}. \qquad (6)$$

For stronger fields, $eEa > \Delta$, the vacancion ion transport is impossible if $|e\mathbf{E}\cdot\mathbf{a}_n| \geq \Delta$ for all $\mathbf{a}_n$. As $\mathbf{E}$ is rotated, electric current appears whenever the condition $|e\mathbf{E}\cdot\mathbf{a}_n| < \Delta$ is met for one of the vectors $\mathbf{a}_n$. If we increase the angle between $\mathbf{E}$ and $\mathbf{a}_n$ (i.e., decrease $|e\mathbf{E}\cdot\mathbf{a}_n|$), the current will increase according to (6) until the saturation (5) is reached. As $|e\mathbf{E}\cdot\mathbf{a}_n|$ decreases further, the current does not depend on $\mathbf{E}$ and is given by (5). Starting from $|e\mathbf{E}\cdot\mathbf{a}_n| \sim T$ the



Dependence of the absolute value of the velocity and of the direction (θ is the angle between u and $\mathbf{a}_1$) on the angle φ between the vectors E and $\mathbf{a}_1$.

current will decrease again according to formula (3), and will vanish at $\mathbf{E}\cdot\mathbf{a}_n = 0$.

It should be noted that in the case of the hexagonal close-packed lattice, the summation over nearest neighbors in all the above expressions must be carried out for both sublattices separately. In weak electric fields $eEa \ll T$, both sublattices enter the sum with the same weight. In strong fields, $eEa \gg T$, the sublattices are not equivalent, because the ion transition probabilities from one sublattice to another and back are different. The statistical weight with which each sublattice is to be summed over is inversely proportional to the probability of ion transition to the other sublattice. In very strong electric fields, $eEa > \Delta$, certain orientations of $\mathbf{E}$ may even give rise to a situation when one-way ion transitions between the nonequivalent sites will be allowed and the inverse processes forbidden. In this case all the ions will become concentrated in one sublattice.

The following reservation must be stipulated for all the results. Although, as pointed out above, the linear dimensions of ions are of the order of the interatomic separation, the "number of particles" in the corresponding complexes can, in general, be large. In this case, the vacancy-ion interaction process is similar to neutron scattering by heavy nuclei. Owing to the large number of interacting particles, the inelastic reaction proceeds in both cases via creation of a long-living compound "nucleus" and has a resonant nature. The cross section of such reactions is described by the well-known Breit-Wigner formula. The constant α in (3) in this case will be a function of temperature determined by the energy dependence of the scattering cross section:

$$\alpha=\frac{(\pi T)^{-3/2}}{2\hbar}\int d\varepsilon\exp\left\{-\frac{\varepsilon-\varepsilon_0}{T}\right\}\int\sigma_n(\mathbf{k})\,dS.$$

However, if the temperature T (and, hence, the energy of the scattered vacancies) is less than the resonance values of the energy, the cross sections are inversely proportional to the velocity, and we are back at the above expressions for the ion mobility.

On the other hand, if the ion structure is simple (e.g., the negative ion can be a charged vacancy-type formation), the above formulae are valid at any temperature.

It should be borne in mind that phonon processes were not accounted for in the above. The latter will be of considerable importance in strong fields $eEa \gtrsim \Delta$ in those cases when the probability of the vacancion ion transport is small. The field dependence of the mobility will become smoothed out by processes associated with vacancy scattering by ions with simultaneous emission or absorption of phonons of energy of order eEa.

2. PROPER MOBILITY

As was pointed out in the Introduction, at low temperatures the ions are delocalized and should be regarded as quasiparticles having a finite bandwidth Δ_i that determines the probability $\Delta_i/\hbar$ of ion tunneling to a neighboring site. As the temperature is lowered the probability of vacancion processes decreases exponentially, and if the condition

$$\frac{\Delta_i}{\Delta}\gg\frac{\alpha\sqrt{\Delta}}{\hbar}\left(\frac{T}{\Delta}\right)^{3/2}e^{-\varepsilon_0/T}$$

is fulfilled, then the main mechanism of ion transport is the ion tunneling proper. If furthermore $T \gg \Theta(\Delta_i/\Theta)^{1/9}$, then the diffusion coefficient equals $\Delta_i a^2/\hbar$, and the ion motion represents a quantum diffusion of localized defects[1]. In the opposite extreme case, $T \ll \Theta(\Delta_i/\Theta)^{1/9}$, the ions represent essentially freely moving quasiparticles. It is this region that we shall consider in detail, because it involves a peculiar external field dependence of the mobility even in very weak electric fields $eEa \ll \Delta_i$[15].

The bandwidth Δ_i is most likely to be small so that we can assume $\Delta_i \ll T$ in the whole accessible temperature range. The band occupation is then practically uniform, and the dependence of the distribution f on quasimomentum $\mathbf{p}$ at equilibrium is given by

$$\frac{\partial f}{\partial\mathbf{p}}=-\frac{f_0}{T}\frac{\partial\varepsilon}{\partial\mathbf{p}},$$

where f_0 is a constant determined by the number of ions per unit volume, and $\epsilon(\mathbf{p})$ is the ion energy spectrum. Since the phonon momenta q are small compared to $|\mathbf{p}| \sim \hbar/a$, the kinetic equation for ions is of the Fokker-Planck form,

$$\frac{\partial f}{\partial\mathbf{p}}e\mathbf{E}=d\frac{\partial}{\partial\mathbf{p}}\left\{\frac{\partial f}{\partial\mathbf{p}}+\frac{f_0}{T}\frac{\partial\varepsilon}{\partial\mathbf{p}}\right\},\tag{7}$$

where d is the diffusion coefficient in momentum space:

$$d\sim\frac{q^2}{\tau}\sim\left(\frac{T}{\Theta}\right)^9\left(\frac{\Theta\hbar}{a^2}\right),$$

and τ is the ion-phonon relaxation time.

By making a Fourier series expansion of both the distribution function and the energy spectrum

$$f(\mathbf{p})=\sum_{\mathbf{a}}f_{\mathbf{a}}e^{i\mathbf{pa}/\hbar},\quad \varepsilon(\mathbf{p})=\sum_{\mathbf{a}}\varepsilon_{\mathbf{a}}e^{i\mathbf{pa}/\hbar},$$

(where $f_{-\mathbf{a}} = f_{\mathbf{a}}^*$, $\epsilon_{-\mathbf{a}} = \epsilon_{\mathbf{a}}^*$, and the summation is over all lattice translation vectors), we obtain from (7)

$$f_{\mathbf{a}}=-\frac{a^2d}{T}f_0\frac{\varepsilon_{\mathbf{a}}}{da^2+ie\hbar\mathbf{Ea}}.$$

The average ion drift velocity is equal to

$$\mathbf{u}=\int\frac{\partial\varepsilon}{\partial\mathbf{p}}f\,d^3\mathbf{p}\Big/\int f_0\,d^3\mathbf{p}=\frac{1}{f_0}\sum_{\mathbf{a}}i\mathbf{a}\varepsilon_{\mathbf{a}}f_{\mathbf{a}}^*$$
$$=-\frac{d}{T}\sum_{\mathbf{a}}\frac{ia^2\mathbf{a}|\varepsilon_{\mathbf{a}}|^2}{da^2-ie\hbar\mathbf{Ea}}.$$

For the case of narrow bands, the dominant harmonics are those corresponding to nearest neighbors. Combining terms with $\mathbf{a}_n$ and $-\mathbf{a}_n$ in the sum, where $e\mathbf{E}\cdot\mathbf{a}_n > 0$, we get

$$\mathbf{u}=\frac{2\Delta_i^2}{T}d\sum_n\frac{e\hbar(\mathbf{Ea}_n)a_n^2\mathbf{a}_n}{(a_n^2d)^2+e^2\hbar^2(\mathbf{Ea}_n)^2}.\tag{8}$$

Here $\Delta_i \equiv |\epsilon_{\mathbf{a}_n}|$ is of the order of the bandwidth. In weak electric fields, the drift velocity is proportional to the field

$$u_i=eB_{ik}E_k,$$

where the mobility tensor is equal to

$$B_{ik}=\frac{2\hbar\Delta_i^2}{Td}\sum_n\frac{a_{ni}a_{nk}}{a_n^2}.$$

In strong fields the drift velocity drops as the field increases

$$\mathbf{u}=\frac{2\Delta_i^2d}{e\hbar T}\sum_n\frac{a_n^2\mathbf{a}_n}{\mathbf{Ea}_n}$$

This has a simple physical meaning. If a constant force eE is applied to a quasiparticle with a spectrum $\epsilon(p)$, then, owing to the finite bandwidth Δ_i, the particle is bound by such a potential and becomes localized[15]. As a consequence, the particle oscillates at a frequency $\omega \sim eEa/\hbar$ in a region of size Δ_i/eE. If $\omega\tau_{tr} \gg 1$, where $\tau_{tr} \sim \hbar^2/a^2d$, there is practically no particle diffusion. Since $\tau_{tr} \propto T^{-9}$ and increases rapidly with decreasing temperature, the nonlinear field dependence of the velocity, described by (8), is appreciable at sufficiently low temperatures even in very weak fields.

We have been discussing the case $eEa \ll \Delta_i$. In the opposite extreme case, $eEa \gg \Delta_i$, the main mechanism of ion transport is tunneling combined with simultaneous phonon scattering, which guarantees energy balance as the ion makes a transition to a neighboring site. If $eEa \ll T$, the appropriate drift velocity is given by[16]

$$u \sim \frac{eEa}{\Theta}\left(\frac{T}{\Theta}\right)^6 \frac{\Delta_i^2 a}{\hbar\Theta}. \qquad (9)$$

For $eEa \gg T$ the drift velocity due to phonon scattering is proportional to the cube of electric field and the fourth power of temperature. However, in this case the ion transitions are much more likely to occur along the field with simultaneous spontaneous emission of a phonon of energy eEa. As usual, the probability of spontaneous emission is proportional to the cube of the emitted phonon frequency and the square of the ion-wave-function overlap integral for neighboring sites, i.e., the square of the bandwidth Δ_i. The drift velocity is therefore, equal to

$$u \sim \left(\frac{eEa}{\Theta}\right)^3 \frac{\Delta_i^2 a}{\hbar\Theta}. \qquad (10)$$

The change-over from formula (9) to (10) occurs at the field values $eEa \sim T^3/\Theta^2$.

3. CONCENTRATION DEPENDENCE OF MOBILITY

The formulae obtained in the first two sections of the present work are also applicable to the case of isotopic impurities if their concentration is small. As the concentration increases, the interaction between impurity atoms becomes appreciable[10,11]. If the temperature is sufficiently low, and the concentration x is so small that the average separation between impurities, $\bar{r} \sim a/x^{1/3}$, much exceeds the impuriton interaction radius R, then the diffusion coefficient D is equal to[10,11]

$$D \sim \Delta_i a^4/\hbar R^2 x. \qquad (11)$$

The interaction radius R is connected with the interaction energy U(r) between two impurity atoms by the relation $U(R) \sim \Delta_i$. $U(r) \sim U_0(a/r)^3$, where U_0 is some characteristic interaction energy, R can be determined from the expression $R \sim a(U_0/\Delta_i)^{1/3}$, i.e., if the band is sufficiently narrow, R exceeds greatly the interatomic separation[17].

In this case, (11) can be rewritten in the following form:

$$D \sim \frac{\Delta_i a^2}{\hbar x}\left(\frac{\Delta_i}{U_0}\right)^{2/3}. \qquad (12)$$

The condition for validity of (12) is given by the inequality $x \ll \Delta_i/U_0 \ll 1$.

Consider a higher concentration domain

$$\Delta_i/U_0 \ll x \ll (\Delta_i/U_0)^{3/4}.$$

In this case, the difference between interaction energies at neighboring sites $a\partial U/\partial r$ is still small compared to Δ_i and the motion of an impurity in the field of other particles is quasiclassical. A given impurity atom moves practically along an equipotential trajectory, because its "kinetic" energy can not change by a quantity larger than Δ_i. Since the potential is due to randomly distributed impurities, such an equipotential trajectory is a random curve with a typical radius of curvature on the order of the average distance $\bar{r}$ between the impurities. The diffusion coefficient is given by the usual gas-kinetic formula $D \sim vl$, the velocity v being of order $\Delta_i a/\hbar$ and the mean free path $l \sim \bar{r}$:

$$D \sim \Delta_i a^2/\hbar x^{1/3}. \qquad (13)$$

The relaxation time T_2 for the region corresponding to (11) and (12) has been found in[10] to be

$$\frac{1}{T_2} \sim \frac{\hbar^3\gamma^4}{R^3 a^4 \Delta_i} x, \qquad (14)$$

where $\hbar\gamma$ is the nuclear magneton. For concentrations $\Delta_i/U_0 \ll x \ll (\Delta_i/U_0)^{3/4}$ a similar consideration leads to the formula

$$\frac{1}{T_2} \sim \frac{\hbar^3\gamma^4}{\Delta_i a^6} x^{5/3}. \qquad (15)$$

Experimental data[9-11] reveal an inversely proportional concentration dependence of the diffusion coefficient at $x \lesssim 10^{-2}$ and of the relaxation time T_2 at $x \lesssim 10^{-3}$. However, the value of R calculated on the basis of (11) and (14) turns out to be of the order of the interatomic separation, whereas the bandwidth Δ_i is surprisingly small, $\Delta_i \sim 10^{-7}$ K. Despite this smallness, the laws (13) and (15) are not observed. This circumstance is hard to interpret at present.

In conclusion, we thank I. M. Lifshitz and A. I. Shal'nikov for useful discussions.

---

[1] A. F. Andreev and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. 56, 2057 (1969) [Sov. Phys.-JETP 29, 1107 (1969)].

[2] R. A. Guyer and L. I. Zane, Phys. Rev. Lett. 24, 660 (1970).

[3] A. I. Shal'nikov, Zh. Eksp. Teor. Fiz. 41, 1056 (1961); 47, 1727 (1964) [Sov. Phys.-JETP 14, 753 (1962; 20, 1161 (1965)].

[4] E. Ifft, L. Mezhov-Deglin, and A. Shal'nikov, Proc. of the 10th Intern. Conf. on Low Temp. Phys., Moscow (1967).

[5] K. O. Keshishev, L. P. Mezhov-Deglin, and A. I. Shal'nikov, ZhETF Pis. Red. 12, 234 (1970) [JETP Lett. 12, 160 (1970)].

[6] G. A. Sai-Halasz and A. J. Dahm, Phys. Rev. Lett. 28, 1244 (1972).

[7] D. Marty and F. I. B. Williams, J. Phys. (Paris) 34, 989 (1973).

[8] R. A. Guyer, R. C. Richardson, and L. I. Zane, Rev. Mod. Phys. 43, 532 (1971).

[9] V. N. Grigor'ev, B. N. Esel'son, V. A. Mikheev, and Yu. E. Shul'man, ZhETF Pis. Red. 17, 25 (1973) [JETP Lett. 17, 16 (1973)].

[10] M. G. Richards, J. Pope, and A. Widom, Phys. Rev. Lett. 29, 708 (1972).

[11] V. N. Grigoriev, V. N. Esel'son, V. A. Mikheev, V. A. Slusarev, M. A. Strzhemechny, and Yu. E. Shulman, J. Low Temp. Phys., 13, 65 (1973).

[12] M. N. Cohen and J. Jortner, Phys. Rev. 180, 238 (1969).

[13] V. B. Shikin, ZhETF Pis. Red. 13, 65 (1971) [JETP Lett. 13, 44 (1971)]; Zh. Eksp. Teor. Fiz. 61, 2053 (1971) [Sov. Phys.-JETP 34, 1095 (1972)].
[14] V. P. Mineev, ZhETF Pis. Red. 17, 161 (1973) [JETP Lett. 17, 112 (1973)].
[15] A. E. Meĭerovich, Zh. Eksp. Teor. Fiz. 67, 744 (1974) [Sov. Phys.-JETP 40, 368 (1975)].
[16] Yu. Kagan and L. A. Maksimov, Zh. Eksp. Teor. Fiz. 65, 622 (1973) [Sov. Phys.-JETP, 38, 307 (1974)].
[17] D. I. Pushkarov, Zh. Eksp. Teor. Fiz. 59, 1755 (1970) [Sov. Phys.-JETP 32, 954 (1971)].

Translated by S. Luryi
169