Magnetic phases of solid He³

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(Submitted June 2, 1977)
Pis'ma Zh. Eksp. Teor. Fiz. **26**, No. 1, 40–44 (5 July 1977)

The possible phases of solid He³ at infralow temperatures are investigated. It is shown that the experimental data (N. B. Kummer *et al.*, Phys. Rev. Lett. **34**, 517, 1975) point to the existence of a ferromagnetic phase containing more than 0.6% ground-state vacancies on the melting point in magnetic fields exceeding 0.2 T.

PACS numbers: 67.80.Jd

Owing to the large amplitude of the zero-point oscillations in quantum crystals of helium, they can contain in principle[1-3] ground-state vacancies whose concentration is finite at zero temperature. In the case of He³ the behavior of the vacancies is quite sensitive to the state of the system of nuclear spins. Ferromagnetic ordering of the spins lowers the energy of the vacancy in comparison with the paramagnetic and the antiferromagnetic states by an appreciable amount of the order of 10 K.[4-6] The existence of null vacancies in He³ is more readily possible therefore only in the ferromagnetic state. Sokoloff and Widom[6] have advance the interesting idea that the experimentally observed[7] rise in the magnetic-ordering temperature[8] with increasing applied magnetic field may be evidence of a transition of He³ into a ferromagnetic state with null vacancies. Their qualitative analysis, however, does not take into account, in particular, the fact that if the nuclear spins are not fully polarized the vacancies should constitute macroscopic formations.[9-10] We consider below the question of the possible magnetic phases of solid He³ and show that allowance for the latter circumstances alters considerably the picture of the phenomenon and leads to a satisfactory explanation of the experimental data of Kummer *et al.*[7]

Consider ferromagnetically ordered He³ at zero temperature. We assume that the minimum vacancy-formation energy is negative. The vacancy energy $e(\mathbf{p})$ near the bottom of the band, as a function of the quasimomentum $\mathbf{p}$, is in this case $\epsilon(\mathbf{p}) = -\epsilon_0 + p^2/2M$, where $\epsilon_0 > 0$ and M is the effective mass of the vacancy. The equilibrium state of the system corresponds to Fermi filling of all the negative levels, i.e., the levels with $p < (2M\epsilon_0)^{1/2}$. The concentration x of the ground-state vacancies and their energy E_v per lattice site are equal to[1]

$$x = \frac{(2M\epsilon_0)^{3/2}}{6\pi^2 N\hbar^3} \quad , \qquad E_v = -\frac{(2M\epsilon_0)^{5/2}}{30\pi^2 N\hbar^3 M} \quad , \tag{1}$$

where N is the number of lattice sites per unit volume. The energy of the ferromagnet is

$$E_f = \theta/2 - \mu H + E_v \ . \tag{2}$$

The first term represents here the exchange energy in the molecular-field approximation (see, e.g.,[11]), and the second is the energy in an external
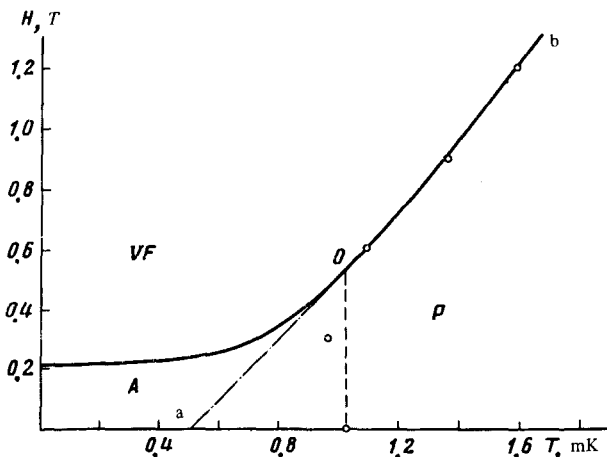
FIG. 1.

magnetic field H (exceeding the saturation field); θ is the constant of the Curie-Weiss law for the susceptibility $\chi \propto (T + \theta)^{-1}$ in the paramagnetic state, and μ is the magnetic moment of the He^3 nucleus. Formula (2) determines in fact the free energy of the ferromagnetic phase inasmuch, as will be shown below, the temperature corrections are small in the considered temperature region.

The free energy of the paramagnetic state, in the molecular field approximation, is equal to

$$F_p(T) = -\frac{\mu^2(H - H_e)^2}{2\theta} - T \ln\left(2 \operatorname{ch} \frac{\mu H_e}{T}\right), \tag{3}$$

where H_e is the effective field and is connected with the external field by the relation

$$\mu H_e = \mu H - \theta \operatorname{th} \frac{\mu H_e}{T}.$$

Formula (3) was written without allowance for the contribution of the vacancies since, as will be shown below, their concentration in the paramagnetic phase is quite small.

By equating (2) and (3), we determine the temperature $T_0(H)$ of the first-order phase transition from the paramagnetic state into the state of a vacancy ferromagnet, a transition accompanied by a jump in the vacancy concentration. The function $T_0(H)$ is shown in Fig. 1 by the aOb curve. The parameters θ and E_v were chosen to satisfy the data of[7], which are represented by the circles on the aOb curve. It turned out that $\theta \approx 0.9$ mK and $|E_v| \approx 0.8$ mK, which yields with the aid of formula (1) a null vacancy concentration $x = 6 \times 10^{-3}(M/m)^{3/5}$ and an energy $\epsilon_0 \approx 0.3(m/M)$ K, where m is the mass of the He^3 atom. (The quantity $(M/m)^{3/5}$ hardly exceeds unity significantly). Since $|E| \sim \theta$, we have in order of magnitude $\epsilon_0 \sim \theta(\Delta/\theta)^{3/5}$, where $\Delta \sim \hbar^2 N^{2/3}/M \sim 10$ K is the width of the energy band of the vacancies in the ferromagnetic phase. The ferromagnetic phase can be regarded as being the ground state with energy (2) at temperatures much

lower than the Fermi energy $T_F \sim \epsilon_0$ of the vacancies and the Curie temperature $T_c \sim x\Delta \sim \Delta(\epsilon_0/\Delta)^{3/2} \sim \theta(\Delta/\theta)^{2/5}$ due to the exchange interaction of the spins via the vacancies. Both conditions are satisfied. The concentration of the vacancies in the paramagnetic phase (cf. [9]) is determined by the relation

$$
x_p \sim \exp\left\{\frac{\epsilon_0}{T} - \frac{5}{6}\frac{\pi^2\hbar^2}{MT}\left[\frac{4MN}{\pi\hbar^2}\left(F_p(0) - F_p(T)\right)\right]^{2/5}\right\} ,
$$

where $F_p(0)$ is equal to the energy of the ferromagnetic phase without allowance for the contribution of the vacancies. On the phase-transition curve we have $|\ln x_p| \sim (\epsilon_0/T)$, whence it follows that at $T \sim \theta$ the vacancy concentration is exponentially small.

Thus, the experimental data in strong fields lead to a value $\theta = 0.9$ mK, thus indicating an antiferromagnetic ordering of the He^3 in the absence of a magnetic field, with a Neel temperature of the order of 1 mK. This transition was indeed observed. [7,8,12]

The free energy of the antiferromagnet, in the molecular field approximation with two sublattices, is equal to

$$
F_a(T) = \frac{\mu^2 H_c^2 T_N}{2(T_N + \theta)^2} - \frac{\mu^2 H^2}{2(T_N+\theta)} - T\ln\left\{2\mathrm{ch}\left(\frac{\mu H_c T_N}{T(T_N+\theta)}\right)\right\} , \tag{4}
$$

where T_N is the Neel temperature which differs from θ in general, and $H_c(T)$ is the critical sublattice-collapse field, defined by the relation

$$
\frac{\mu H_c}{T_N + \theta} = \mathrm{th}\left\{\frac{\mu H_c T_N}{T(T_N+\theta)}\right\} .
$$

It is assumed that H greatly exceeds the sublattice flopping field. A continuation of the first-order phase-transition line into the region of low temperatures (section Ob of the solid curve in Fig. 1) was determined from the condition that expressions (4) and (2) be equal at $T_N = 1.03$ mK (see[7]). The dashed line in Fig. 1 is the curve of the second-order phase transition from the antiferromagnetic to the paramagnetic state $H = H_c(T)$.

We arrive thus at the conclusion that the vacancy ferromagnetic phase (VF) should exist only in magnetic fields stronger than 0.2 T. The tricritical point 0 corresponds to $H \approx 0.5$ T and $T \approx T_N$.

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