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He^3–He^4 solutions and other low-density Fermi liquids in a magnetic field

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A theory is proposed for a Fermi liquid of low density, such as a solution of He^3 in superfluid He^4, in an arbitrary magnetic field. The characteristics of the solution are determined by an expansion in the concentration $x^{1/3}$ and are specified, in a field that is not too strong, by a single parameter, the s-scattering length. At almost total polarization of the spin system, the main contribution is already made by p-scattering. Values are obtained for the thermodynamic and hydrodynamic quantities and for the propagation velocity of the low-frequency oscillations (transverse spin waves, high-frequency first sound, and coupled spin-sound modes that exist in weak magnetic fields). The most noticeable are the magnetokinetic effects connected with the appreciable growth of the fermion mean free path and of the kinetic coefficients.

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1. INTRODUCTION

The connection between the Fermi-liquid function of a system in an external magnetic field H and its value in the absence of a field has been established only for weak magnetic fields $\beta H \ll T_F$ (β is the magnetic moment of the fermions and T_F is the degeneracy temperature), when the corrections to the f-function are small to the extent that the field is weak. A consistent calculation of the Landau f-function[1] in the absence of a magnetic field can be carried out in the case of a low-density Fermi system with the aid of an expansion in $x^{1/2}$ ($x \ll 1$ is the concentration of the fermions). All the properties of such systems are then specified by a single interaction parameter–the s-scattering length a of the Fermi particles.[2-7]

In this paper we propose a theory of an uncharged low-density Fermi liquid in an arbitrary magnetic field with account taken also of the possible presence of a superfluid background. The analysis is based on the fact that for low-density Fermi systems the expansion in the interaction coincides formally with the expansion of all the quantities in powers of $x^{1/3}$. The interaction of the bare particles in the nonrelativistic approximation does not depend on the spin, and the scattering amplitude does not depend on the magnetic field. This makes it possible to relate, in first approximation in the interaction, the f-function of the isotropic Fermi liquid with the amplitude of particle scattering in the absence of a field.

In an isotropic Fermi liquid of spin-1/2 particles, the Fermi surfaces of quasiparticles with differently oriented spins constitute two Fermi spheres whose radii are determined by the degree of polarization of the spin system, i.e., by the value of the magnetic field. In the course of the interaction, the particles with the two polarizations remain on their Fermi surfaces. If the particle interaction energy decreases rapidly enough with increasing distance between them, then the scattering amplitude of the slow particles, and consequently the low-density Fermi-liquid function, are determined by the s-scattering.

Since the fermions are identical, only particles with oppositely directed spins interact in the s-scattering. In strong magnetic fields, when practically all the spins have the same orientation, the Fermi-liquid interaction is already determined by the p-scattering.[8] This weakens considerably the interaction in strong fields and alters substantially the concentration dependences of all the thermodynamic quantities. For this reason, the values of the kinetic coefficients, such as viscosity, thermal conductivity, and others, increase strongly with increasing magnetic field (with increasing degree of polarization of the fermions).[8]

The most typical example of a low-density Fermi liquid is the degenerate solution of He^3 in superfluid He^4, in which an important role is played also by the interaction of the fermions with the superfluid background that moves with velocities v_s.[9] The calculations that follow have therefore been performed for degenerate He^3–HeII solutions.

In the next section of the paper we derive general relations that connect the density matrix, the energy spectrum, and the f-function of a Fermi system in the magnetic field in the presence also of a superfluid background. We consider next the thermodynamic properties of a partially as well as fully polarized Fermi liquid (Sec. 3), high-frequency spin and acoustic oscillations (Sec. 4), and magnetokinetic phenomena (Sec. 5). In the Conclusion we present numerical estimates, and discuss the limits of applicability of the theory and the experimental consequences of the results. Some preliminary results were published before.[8]

2. EXCITATION ENERGY

The excitation energy for a Fermi system of He^3 quasiparticles in superfluid He^4 in a constant magnetic field $\mathbf{H}$ depends on the value of the spin projection on the spin direction, and in the absence of superfluid motion can be represented in the form

$$\varepsilon_{\alpha\beta}^{(0)}(\mathbf{p})=\varepsilon_0(\mathbf{p})\delta_{\alpha\beta}-B(\mathbf{p})\sigma_{\alpha\beta}\mathbf{H}, \tag{2.1}$$

where $\mathbf{p}$ is the excitation momentum, α and β are spin indices, $\sigma_{\alpha\beta}$ are Pauli matrices, and the functions $\epsilon_0(\mathbf{p})$ and $B(\mathbf{p})$ depend in the general case on $\mathbf{H}^2$. The equilibrium single-particle density matrix for a polarized Fermi system is of the form

$$n_{\alpha\beta}^{(0)}(\mathbf{p})={}^1/_2 n(\mathbf{p})\delta_{\alpha\beta}+{}^1/_2\rho(\mathbf{p})\sigma_{\alpha\beta}. \tag{2.2}$$

The values $n(\mathbf{p})\equiv n_{\alpha\alpha}^{(0)}$ and $\rho(\mathbf{p})\equiv\sigma_{\alpha\beta}n_{\beta\alpha}^{(0)}$ (the summation over dummy indices is used here and throughout) are determined from the condition that at equilibrium the occupation numbers $n_0^{\pm}(\mathbf{p})$ of particles with momentum and spin projection ± 1/2 on the $\mathfrak{M}=\mathbf{H}/H$ axis are given by a stepwise Fermi distribution function n_F:

$$\begin{gathered}n_0^{\pm}(\mathbf{p})=n_F(\varepsilon_0\mp BH),\\ n(\mathbf{p})=n_0^+(\mathbf{p})+n_0^-(\mathbf{p}),\quad \rho(\mathbf{p})=[n_0^+(\mathbf{p})-n_0^-(\mathbf{p})]\mathfrak{M}.\end{gathered} \tag{2.3}$$

The excitation energy is a functional of the distribution function[1]

$$\delta\varepsilon_{\alpha\beta}(\mathbf{p})=\frac{1}{2}\int f_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}')\delta n_{\nu\mu}(\mathbf{p}')d\Gamma'-\beta\sigma_{\alpha\beta}\delta\mathbf{H}, \tag{2.4}$$

where $d\Gamma=2d^3p/(2\pi\hbar)^3$, and $\delta n_{\alpha\beta}$ is the deviation of the density matrix from the equilibrium value (2.2). The Fermi-liquid function $f_{\alpha\beta,\,\mu\nu}(\mathbf{p},\mathbf{p}')$ can be written in the exchange approximation in the sum

$$\begin{aligned}f_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}')&=\psi(\mathbf{p},\mathbf{p}')\delta_{\alpha\beta}\delta_{\mu\nu}+\zeta(\mathbf{p},\mathbf{p}')\sigma_{\alpha\beta}\sigma_{\mu\nu}\\&+[\varphi(\mathbf{p},\mathbf{p}')\sigma_{\alpha\beta}\delta_{\mu\nu}+\varphi(\mathbf{p}',\mathbf{p})\sigma_{\mu\nu}\delta_{\alpha\beta}]\mathfrak{M}+\xi(\mathbf{p},\mathbf{p}')(\sigma_{\alpha\beta}\mathfrak{M})(\sigma_{\mu\nu}\mathfrak{M}).\end{aligned} \tag{2.5}$$

In analogy with[10], we can use (2.2)–(2.5) to determine the function $B(\mathbf{p})$

$$B(\mathbf{p})=\beta-\frac{1}{2H}\int\zeta(\mathbf{p},\mathbf{p}')[n_0^+(\mathbf{p}')-n_0^-(\mathbf{p}')]d\Gamma'. \tag{2.6}$$

Following onset of superfluid motion with velocity $\mathbf{v}_s$, in a reference frame in which there is no superfluid motion, the change of the energy can be represented in an approximation linear in $\mathbf{v}_s$ by

$$\delta\varepsilon_{\alpha\beta}={}^1/_2\{X(\mathbf{p})\delta_{\alpha\beta}+Y(\mathbf{p})\sigma_{\alpha\beta}\}\mathbf{p}\mathbf{v}_s. \tag{2.7}$$

To determine the functions $X(\mathbf{p})$ and $Y(\mathbf{p})$ we use an equation (see[7,11]) obtained by Khalatnikov with the aid of Galilean transformations; this equation specifies the change of the energy in the reference frame in which the superfluid background is at rest:

$$\delta\varepsilon_{\alpha\beta}(\mathbf{p})=\mathbf{p}\mathbf{v}_s\delta_{\alpha\beta}+\frac{1}{2}\int f_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}')\delta n_{\nu\mu}^{(c)}(\mathbf{p}')d\Gamma'. \tag{2.8}$$

The equilibrium density matrix (2.2), (2.3) is a function of the variables $\epsilon^{\pm}\equiv(\epsilon_{\alpha\alpha}\pm\epsilon_{\alpha}\sigma_{\beta\,\alpha}\mathfrak{M})/2=\epsilon_0\mp BH$. The change of the density matrix following the onset of the superfluid motion is

$$\delta n_{\alpha\beta}^{(0)}=\frac{1}{2}\left\{\frac{\partial n_{\alpha\beta}^{(0)}}{\partial\varepsilon^+}(X+\mathfrak{M}Y)+\frac{\partial n_{\alpha\beta}^{(0)}}{\partial\varepsilon^-}(X-\mathfrak{M}Y)\right\}\mathbf{p}\mathbf{v}_s. \tag{2.9}$$

Substitution of (2.7) and (2.9) in (2.8) results in a system of integral equations for the functions $X(\mathbf{p})$ and $Y(\mathbf{p})$. This system can be easily solved for a isotropic Fermi liquid with equilibrium density matrix (2.2), (2.3), but the solutions are quite unwieldy and will not be needed in direct form. All they need is the values of $\delta\epsilon_{\alpha\beta}(\mathbf{p})$ for the particles whose momenta $\mathbf{p}_+$ and $\mathbf{p}_-$ lie on the Fermi spheres $\epsilon^+(\mathbf{p}_+)=\mu_3$, $\epsilon^-(\mathbf{p}_-)=\mu_3$ (μ_3 is the chemical potential of He^3 in the solution). The quantities $2A^{\pm}\equiv X(\mathbf{p}_{\pm})+\mathfrak{M}Y(\mathbf{p}_{\pm})$ are determined from Eqs. (2.7)–(2.9):

$$A^{\pm}=(Q^{\pm}-R^{\mp})/(R^+R^--Q^+Q^-), \tag{2.10}$$

where the functions $R^{\pm}$ and $Q^{\pm}$ are specified by linear combinations of the first harmonics of the expansion of the Fermi-liquid function (2.5) in Legendre polynomials

$$\begin{aligned}R^{\pm}&=-1-\frac{1}{2}\left(\frac{d\Gamma}{d\varepsilon}\right)_{\pm}\int\frac{do'}{4\pi}\frac{\mathbf{p}_{\pm}\mathbf{p}_{\pm}'}{p_{\pm}^2}\{\psi(\mathbf{p}_{\pm},\mathbf{p}_{\pm}')+\zeta(\mathbf{p}_{\pm},\mathbf{p}_{\pm}')\\&\qquad\pm\varphi(\mathbf{p}_{\pm},\mathbf{p}_{\pm}')\pm\varphi(\mathbf{p}_{\pm}',\mathbf{p}_{\pm})+\xi(\mathbf{p}_{\pm},\mathbf{p}_{\pm}')\},\\Q^{\pm}&=-\frac{1}{2}\left(\frac{d\Gamma}{d\varepsilon}\right)_{\mp}\int\frac{do'}{4\pi}\frac{\mathbf{p}_{\pm}\mathbf{p}_{\mp}'}{p_{\pm}^2}\{\psi(\mathbf{p}_{\pm},\mathbf{p}_{\mp}')-\zeta(\mathbf{p}_{\pm},\mathbf{p}_{\mp}')\pm\varphi(\mathbf{p}_{\pm},\mathbf{p}_{\mp}')\\&\qquad\mp\varphi(\mathbf{p}_{\mp}',\mathbf{p}_{\pm})-\xi(\mathbf{p}_{\pm},\mathbf{p}_{\mp}')\}.\end{aligned} \tag{2.11}$$

Inasmuch as a transition from the reference frame in which the superfluid background is at rest to the laboratory frame causes the excitation momentum to be transformed like $\mathbf{p}=\mathbf{p}'+m_3\mathbf{v}_s$ (m_3 is the set mass of the He^3 atom), the energy spectrum of the excitations in the laboratory frame is

$$\varepsilon_{\alpha\beta}(\mathbf{p})=\varepsilon_0(\mathbf{p}-m_3\mathbf{v}_s)\delta_{\alpha\beta}-B(\mathbf{p}-m_3\mathbf{v}_s)\sigma_{\alpha\beta}\mathbf{H}+\delta\varepsilon_{\alpha\beta}(\mathbf{p}-m_3\mathbf{v}_s)+{}^1/_2 m_3\mathbf{v}_s^2\delta_{\alpha\beta},$$

where $\delta\epsilon_{\alpha\beta}(\mathbf{p})$ is given by

$$\delta\varepsilon_{\alpha\beta}=\{{}^1/_2(A^++A^-)\delta_{\alpha\beta}+{}^1/_2(A^+-A^-)\mathfrak{M}\sigma_{\alpha\beta}\}\mathbf{p}\mathbf{v}_s.$$

The final expressions for the l.s. energy of the excitations and for the equilibrium density matrix, in the approximation linear in $\mathbf{v}_s$, are

$$\begin{gathered}\varepsilon_{\alpha\beta}^{(0)}=\varepsilon_0\delta_{\alpha\beta}-B\sigma_{\alpha\beta}\mathbf{H}+(\delta m)_{\alpha\gamma}(1/m)_{\gamma\beta}\mathbf{p}\mathbf{v}_s,\\ n_{\alpha\beta}=n_{\alpha\beta}^{(0)}+\left[\frac{(\delta m)_+}{m_+}\frac{\partial n_{\alpha\beta}^{(0)}}{\partial\varepsilon^+}+\frac{(\delta m)_-}{m_-}\frac{\partial n_{\alpha\beta}^{(0)}}{\partial\varepsilon^-}\right]\mathbf{p}\mathbf{v}_s.\end{gathered} \tag{2.12}$$

Here $n_{\alpha\beta}^{(0)}$ is defined by formulas (2.2) and (2.3), $(\delta m)_{\alpha\beta}=M_{\alpha\beta}-m_3\delta_{\alpha\beta}$, $(\delta m)_{\pm}=M_{\pm}-m_3$, and $M_{\alpha\beta}$ is the effective-mass spinor of a single bare quasiparticle with energy spectrum $\xi_{\alpha\beta}$ in immobile superfluid He^4:

$$\begin{gathered}M_{\alpha\beta}={}^1/_2(M_++M_-)\delta_{\alpha\beta}+{}^1/_2(M_+-M_-)\mathfrak{M}\sigma_{\alpha\beta},\\ \mathscr{E}_{\alpha\beta}={}^1/_2(\mathscr{E}_++\mathscr{E}_-)\delta_{\alpha\beta}+{}^1/_2(\mathscr{E}_+-\mathscr{E}_-)\mathfrak{M}\sigma_{\alpha\beta},\end{gathered}$$

and the eigenvalues $\xi_{\pm}$ and

$$M_{\pm}^{-1}=\frac{1}{p_{\pm}}\frac{\partial\mathscr{E}_{\pm}}{\partial p_{\pm}} \tag{2.13}$$

have the meaning of the energy and effective mass of the bare quasi-particle with definite spin projection on the field direction. The eigenvalues of the effective-mass spinor $(1/m)_{\alpha\beta}$ of the Fermi-liquid quasiparticle

$$\left(\frac{1}{m}\right)_{\alpha\beta}=\frac{1}{2}\left(\frac{1}{m_+}+\frac{1}{m_-}\right)\delta_{\alpha\beta}+\frac{1}{2}\left(\frac{1}{m_+}-\frac{1}{m_-}\right)\mathfrak{M}\sigma_{\alpha\beta}, \qquad \frac{1}{m_\pm}=\frac{1}{p_\pm}\frac{\partial\varepsilon^\pm}{\partial p_\pm}$$

are equal, in first order in $\mathbf{v}_s$, to

$$m_\pm=M_\pm/A^\pm. \tag{2.14}$$

Formulas (2.14) are exact for isotropic Fermi liquids in the absence of superfluid background with a quadratic dispersion law of bare particles. In this case $M_+=M_-$ should be taken to mean the true mass of the real particles that make up the liquid.

3. FERMI-LIQUID FUNCTION AND THERMODYNAMIC PROPERTIES

The energy spectrum of a bare He^3 quasiparticle in superfluid He^4 is given by

$$\mathscr{E}_{\alpha\beta}(\mathbf{p})=\left[-\Delta+\frac{p^2}{2M}\left(1-\frac{\nu p^2}{p_c^2}\right)\right]\delta_{\alpha\beta}-\beta\sigma_{\alpha\beta}\mathbf{H}, \tag{3.1}$$

where the energy gap $\Delta\sim 2.8$K, $M\approx 2.3m_3$, $p_c=m_4s_0$, m_4 is the mass of the He^4 atom, s_0 is the speed of sound in pure He^4, and the dimensionless parameter ν is quite small according to the experimental data.[12,13] From (2.13) and (3.1) we have

$$M_\pm=M/(1-2\nu p_\pm^2/p_c^2).$$

A. Partially polarized Fermi systems

It was noted in the Introduction that the properties of an incompletely polarized Fermi liquid of low density are determined only by a single parameter–the s-scattering length a of the quasiparticles. For the He^3–He^4 solution we have $a\approx 1.5\times 10^{-8}$ cm.[9]

In first-order perturbation theory, the total energy of the system is equal to[2-4,9]

$$E=E_4^{(0)}+\frac{1}{2}\int\mathscr{E}_{\alpha\beta}(\mathbf{p})n_{\beta\alpha}^{(0)}(\mathbf{p})\,d\Gamma$$
$$+\frac{\pi a\hbar^2}{M}\int\{n_{\alpha\alpha}^{(0)}(\mathbf{p})n_{\beta\beta}^{(0)}(\mathbf{p}')-n_{\alpha\beta}^{(0)}(\mathbf{p})n_{\mu\nu}^{(0)}(\mathbf{p}')\sigma_{\alpha\beta}\sigma_{\mu\nu}\}\,d\Gamma\,d\Gamma'. \tag{3.2}$$

Here $E_4^{(0)}$ is the energy of pure He^4. There is no need to take into account in (3.2) the term of (3.1) with ν, inasmuch as in the case of s-scattering it is significant only in higher orders of perturbation theory. For the second-order perturbation-theory correction, the results are quite cumbersome and are given in the Appendix.

The Fermi-liquid function is equal to (cf.[4])

$$f_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}')=\left[\frac{\delta^2E}{\delta n_{\beta\alpha}(\mathbf{p})\,\delta n_{\nu\mu}(\mathbf{p}')}\right]=\frac{2\pi a\hbar^2}{M}(\delta_{\alpha\beta}\delta_{\mu\nu}-\sigma_{\alpha\beta}\sigma_{\mu\nu}). \tag{3.3}$$

Direct calculation of the energy (3.2) leads to the result

$$E=E_4^{(0)}-N_3\Delta+\frac{3}{10M}(p_+^2N_++p_-^2N_-)-\beta H(N_+-N_-)+\frac{4\pi a\hbar^2}{M}N_+N_-, \tag{3.4}$$

where N_3 is the number of He^3 atoms per unit volume of the solution, $N_\pm=p_\pm^3/(6\pi^2\hbar^3)$ is the total number of particles with different spin orientations per unit volume. The expressions for the energy (3.4) and for the Fermi-liquid function (3.3) make it possible to determine all the thermodynamic properties of the solution.

The quantities $N_\pm$ are calculated with the aid of the equation $(\partial E/\partial N_+)_{N_3,H}=0$:

$$N_\pm=\tilde{N}_\pm\pm 2\left(\frac{6}{\pi}\right)^{1/3}a(\tilde{N}_+\tilde{N}_-)^{1/3}\frac{\tilde{N}_+-\tilde{N}_-}{\tilde{N}_+^{1/3}+\tilde{N}_-^{1/3}},$$

and the chemical potential of He^3 is equal to

$$\mu_3=\mu_0+\frac{4\pi a\hbar^2}{M}(\tilde{N}_+\tilde{N}_-)^{1/3}\frac{\tilde{N}_+^{2/3}+\tilde{N}_-^{2/3}}{\tilde{N}_+^{1/3}+\tilde{N}_-^{1/3}}, \tag{3.5}$$

where $\tilde{N}_\pm(\mathbf{H})$ are the numbers of particles with different spin orientation per unit volume of the ideal Fermi gas, and $\mu_0(\mathbf{H})$ is the chemical potential of the fermions in the absence of interaction. Hereafter we shall sometimes find it convenient to use in place of the variables $N_\pm, \tilde{N}_\pm, H$ the quantities

$$\chi_\pm=p_\pm/p_0=(2N_\pm/N_3)^{1/3},\quad \tilde{\chi}_\pm=(2\tilde{N}_\pm/N_3)^{1/3},$$
$$\mathscr{H}=2\beta H/T_F,\quad p_0=(3\pi^2N_3)^{1/3}\hbar,\quad T_F=2^{2/3}p_0^2/2m_+.$$

Here p_0 is the limiting Fermi momentum in the absence of the field, T_F is the degeneracy temperature of the completely polarized solution. In terms of these variables, the equations that determine the concentrations $\tilde{N}_\pm$ for an ideal Fermi gas are

$$\tilde{\chi}_+^2-\tilde{\chi}_-^2=2^{2/3}\mathscr{H},\quad \tilde{\chi}_+^3+\tilde{\chi}_-^3=2, \tag{3.6}$$

and the chemical potential $\mu_0(\mathbf{H})$ is equal to

$$\mu_0(\mathbf{H})=\frac{p_0^2}{M}(\tilde{\chi}_++\tilde{\chi}_-)^{-1}-\beta H\frac{\tilde{\chi}_+-\tilde{\chi}_-}{\tilde{\chi}_++\tilde{\chi}_-}. \tag{3.7}$$

With the aid of the expressions (3.5) and (3.7) for the chemical potential μ_3, confining ourselves to the terms linear in the concentration, we can easily determine the velocity of second sound in the solution:

$$s_2^2=\frac{N_3}{M}\frac{\partial\mu_3}{\partial N_3}-\frac{m_4}{M}\alpha^2s_0^2\frac{N_3}{N_4},$$

where $\alpha=-(N_4/m_4s_0^2)\partial\Delta/\partial N_4\approx 1.28$[14] and N_4 is the number of He^4 per unit volume. Figure 1 shows a plot of the function $s_2^2(\mathscr{H})$ in the principal approximation in the concentration:

$$\frac{s_2^2(\mathscr{H})}{s_2^2(0)}=\frac{2}{\tilde{\chi}_++\tilde{\chi}_-},\qquad s_2(0)=\frac{p_0}{M\sqrt{3}}. \tag{3.8}$$

It is easy also to obtain the chemical potential of He^4 in the solution:

$$\mu_4=\mu_4^{(0)}-N_3\frac{\partial\Delta}{\partial N_4}-\frac{3\gamma}{10MN_4}(p_+^2N_++p_-^2N_-) \tag{3.9}$$

and the magnetic susceptibility of the solution:

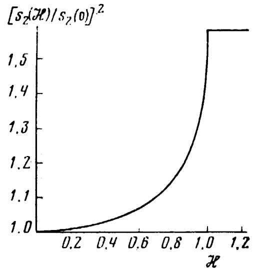

FIG. 1. Relative change of the square of the second-sound velocity $[s_2(\mathcal{H})/s_2(0)]^2$ as a function of the magnetic field. $s_2 = 2^{1/2} s_2(0)$ at $\mathcal{H} \geq 1$.

$$\chi = \beta \frac{\partial}{\partial H}(N_+ - N_-),$$

where

$$\gamma = \partial(\ln M)/\partial(\ln N_4) \approx 1.25\ [4].$$

Analogously, we can obtain in the usual manner[7, 9] the values of the remaining thermodynamic quantities in magnetic fields.

B. Fermi systems with high degree of polarization

Although the Fermi-liquid function (3.3) is formally independent of the field, the terms that describe the interaction in all the thermodynamic quantities obtained above tend to zero with increasing zero, owing to the decrease in the number of particles having spins opposite to the field direction. The reason is that only interaction of particles with opposite spins is effective in s-scattering. If the spin system is almost completely polarized, the properties of a low-density Fermi system are determined mainly by p-scattering. In this case an important role is played only by interaction of quasiparticles on a Fermi surface of radius

$$p_+ \approx \hbar (6\pi^2 N_3)^{1/3} = 2^{1/3} p_0 = p_F.$$

The amplitude of scattering of too slow bare particles with momenta $\mathbf{p}_1$ and $\mathbf{p}_2 (p_1 = p_2 = p_F)$ in the c.m.s. is determined by the rotation angle φ relative to the momentum $\mathbf{p} = (\mathbf{p}_1 - \mathbf{p}_2)/2$:

$$\operatorname{Re} f(\mathbf{p}, \mathbf{p}') = \frac{4\pi\hbar^2}{M}\left(-a + \frac{3bp^2}{\hbar^2}\cos\varphi\right), \tag{3.10}$$

where $\mathbf{p}' = (\mathbf{p}_1' - \mathbf{p}_2')/2$ is the relative momentum of the scattered particles $(p_1' = p_2' = p_F, p' = p)$. The constant b is of the order of the glass-kinetic volume of the atom, $b \sim a^3$ (the quantity b is the equivalent of $B/24\pi$ in[8]).

Taking into account the identity of the fermions, the vertex part, in first order, is equal to

$$\Gamma(\mathbf{p}, \mathbf{p}') = \operatorname{Re}[f(\varphi) - f(\pi - \varphi)] = 24\pi b p^2 M^{-1} \cos\varphi.$$

For the self-energy part ϵ we use the Galitskiĭ equation[5]

$$\Sigma = \frac{1}{2}\int d\Gamma' n_+(\mathbf{p}')\Gamma(\mathbf{q}, \mathbf{q}) = \frac{18\pi b N_3}{M}\left(\frac{p^2}{3} + \frac{p_F^2}{5}\right), \tag{3.11}$$

where $\mathbf{q} = (\mathbf{p} - \mathbf{p}')/2$. The pole of the single-particle Green's function $G(\mathbf{p})$

$$G^{-1}(\mathbf{p}) = \varepsilon(\mathbf{p}) + \Delta - \frac{p^2}{2M}\left[1 - \nu\frac{p^2}{p_c^2}\right] - \beta H - \Sigma,$$

determines the spectrum of the excitations

$$\varepsilon(\mathbf{p}) = -\Delta + \frac{p^2}{2M}\left[1 - \nu\frac{p^2}{p_c^2} + 12\pi b N_3\right] + \frac{36\pi p_F^2}{10M} b N_3 - \beta H,$$

the chemical potential

$$\mu_3 = \varepsilon(\mathbf{p}_F) = -\Delta + \frac{p_F^2}{2M}\left[1 - \nu\frac{p_F^2}{p_c^2} + \frac{96\pi}{5} b N_3\right] - \beta H, \tag{3.12}$$

and the effective mass of the excitations

$$\frac{1}{m_+} = \frac{1}{p_F}\left(\frac{\partial\varepsilon}{\partial p}\right)_{p=p_F}; \qquad \frac{m_+}{M} = 1 + 2\nu\frac{p_F^2}{p_c^2} - 12\pi b N_3.$$

The Fermi-liquid function $f(\theta)$ (θ is the angle between the vectors $\mathbf{p}_1$ and $\mathbf{p}_2$) is determined by the forward-scattering amplitude $\Gamma(\mathbf{p}, \mathbf{p})$[11]:

$$f(\theta) = \frac{24\pi p_F^2}{M} b \sin^2\frac{\theta}{2}. \tag{3.13}$$

Expressions (3.12) and (3.13) determine all the thermodynamic properties of polarized solutions. Thus, for the total energy we have

$$E = E_4^{(0)} - \Delta N_3 + \frac{3p_F^2}{10M} N_3\left[1 - \frac{5\nu}{7}\frac{p_F^2}{p_c^2} + 12\pi b N_3\right] - \beta H N_3 \tag{3.14}$$

The propagation velocities of the hydrodynamic oscillations are determined formally by the same equations as in the absence of a field,[9] with account taken of relations (3.12)–(3.14). The small terms proportional to $N_3^{5/3}$ in the expressions for the velocities of the first and second sounds[9] increase somewhat in this case, and the second-sound velocity s_2 increases by approximately $2^{1/3}$ times and takes the form$^{1)}$

$$\begin{aligned} s_2^2 = \frac{p_F^2}{3M^2}\Bigg[1 - 4\nu\frac{p_F^2}{p_c^2} + 3\alpha\frac{N_3}{N_4}\left(\gamma + 2\alpha\nu\frac{M}{m_4}\right) \\ - \left(\alpha + \frac{M}{m_4} - \frac{m_3}{m_4}\right)^2 \frac{N_3 m_4}{N_4 M} + 48\pi b N_3\Bigg] - \alpha s_0^2 \frac{N_3 m_4}{N_4 M}. \end{aligned} \tag{3.15}$$

The expression for $s_2(\mathcal{H})$(3.8) as $\mathcal{H} \to 1$ coincides with the term principal in the concentration in (3.15).

4. SPIN WAVES

Since the Fermi-liquid interaction of the impurity quasiparticles is small and $a < 0$, it follows that spin waves can exist in the degenerate solution, but propagation of waves of the zero-sound types is impossible.[9] With decreasing temperature, the relaxation time τ of the impurity excitations increases strongly, and the first sound in a solution with $\omega\tau \gg 1$ is described not by the hydrodynamics equations, but by a collisionless kinetic equation. The high-frequency first sound constitutes oscillations of He^4, in which the impurity quasiparticles also participate because of the interaction of the He^3 atoms with the superfluid background.

The high-frequency oscillations of the solution are described by the collisionless kinetic equation for the single-particle density matrix

$$\frac{\partial n_{\alpha\beta}}{\partial t}+\frac{i}{\hbar}[\hat{\varepsilon},\hat{n}]_{\alpha\beta}+\frac{1}{2}\left(\frac{\partial\varepsilon_{\alpha\gamma}}{\partial\mathbf{p}}\nabla n_{\gamma\beta}+\nabla n_{\alpha\gamma}\frac{\partial\varepsilon_{\gamma\beta}}{\partial\mathbf{p}}\right)-\frac{1}{2}\left(\nabla\varepsilon_{\alpha\gamma}\frac{\partial n_{\gamma\beta}}{\partial\mathbf{p}}+\frac{\partial n_{\alpha\gamma}}{\partial\mathbf{p}}\nabla\varepsilon_{\gamma\beta}\right)=0, \tag{4.1}$$

by the continuity equation

$$\frac{\partial}{\partial t}\left(m_4N_4+\frac{1}{2}m_3\int n_{\alpha\alpha}\,d\Gamma\right)+\operatorname{div}\left(m_4N_4\mathbf{v}_s+\frac{1}{2}\int\mathbf{p}n_{\alpha\alpha}\,d\Gamma\right)=0 \tag{4.2}$$

and by the superfluid-motion equation

$$\frac{\partial\mathbf{v}_s}{\partial t}+\frac{1}{m_4}\nabla\frac{\partial}{\partial N_4}\left(E_0+\frac{1}{2}\int\varepsilon_{\alpha\beta}\delta n_{\beta\alpha}\,d\Gamma\right)=0, \tag{4.3}$$

where $[\hat{\epsilon},\hat{n}]_{\alpha\beta}$ is the commuter of the spin matrices and E_0 is the total equilibrium energy of the system. The deviation $\delta n_{\alpha\beta}$ of the density matrix from the equilibrium value is given in first-order perturbation theory by the

$$n_{\alpha\beta}=n_{\alpha\beta}^{(0)}+\frac{\mathbf{p}\mathbf{v}_s}{2}\frac{\delta m}{M}\left[\left(\frac{\partial n_+}{\partial\varepsilon^+}+\frac{\partial n_-}{\partial\varepsilon^-}\right)\delta_{\alpha\beta}+\left(\frac{\partial n_+}{\partial\varepsilon^+}-\frac{\partial n_-}{\partial\varepsilon^-}\right)\sigma_{\alpha\beta}\mathfrak{M}\right]+\delta n_{\alpha\beta},$$

$$\delta n_{\alpha\beta}=\nu(\mathbf{p})\delta_{\alpha\beta}+\lambda(\mathbf{p})\sigma_{\alpha\beta}. \tag{4.4}$$

According to (2.1), (2.4), and (2.12) the energy $\epsilon_{\alpha\beta}$ of the excitations is

$$\varepsilon_{\alpha\beta}=\varepsilon_{\alpha\beta}^{(0)}+\frac{\partial\varepsilon_{\alpha\beta}^{(0)}}{\partial N_4}\delta N_4+\frac{\delta m}{M}\mathbf{p}\mathbf{v}_s\delta_{\alpha\beta}+\psi\delta_{\alpha\beta}\int\nu\,d\Gamma+\zeta\sigma_{\alpha\beta}\int\lambda\,d\Gamma, \tag{4.5}$$

where account is taken of the fact that, in first order in the interaction, the f-function (3.3) does not depend on the momentum. We seek the small perturbations of $v,\lambda,\mathbf{v}_s,\delta N_4$ in the form $\exp(i\omega t-i\mathbf{k}\cdot\mathbf{r})$. When (4.4) and (4.5) are taken into account, the linearized kinetic equation (4.1) breaks up into a scalar component

$$\nu(\omega-\mathbf{kv})-\mathbf{kv}\delta_+\left[\frac{\omega}{\mathbf{kv}}\frac{\delta m}{M}\mathbf{p}\mathbf{v}_s+\frac{\partial\varepsilon_0}{\partial N_4}\delta N_4+\psi\int\nu\,d\Gamma\right]+\mathbf{kv}\delta_-\left[H\frac{\partial B}{\partial N_4}\delta N_4-\zeta\mathfrak{M}\int\lambda\,d\Gamma\right]=0, \tag{4.6}$$

and a vector component

$$\lambda(\omega-\mathbf{kv})-(\mathbf{kv})\mathfrak{M}\delta_-\left[\frac{\omega}{\mathbf{kv}}\frac{\delta m}{M}\mathbf{p}\mathbf{v}_s+\frac{\partial\varepsilon_0}{\partial N_4}\delta N_4+\psi\int\nu\,d\Gamma\right]+\mathbf{kv}\delta_+\left[H\frac{\partial B}{\partial N_4}\delta N_4\mathfrak{M}-\zeta\int\lambda\,d\Gamma\right]-\frac{2i}{\hbar}B[\mathbf{H}\lambda]-\frac{i}{\hbar}\zeta\left[\rho\int\lambda\,d\Gamma\right]=0, \tag{4.7}$$

where

$$\mathbf{v}=\partial\varepsilon_0/\partial\mathbf{p},\qquad \delta_\pm=(1/2)[\delta(\varepsilon^+)\pm\delta(\varepsilon^+)],\qquad \delta m=M-m_3.$$

A. Transverse magnons

We choose as the z axis the direction of the magnetic field $\mathfrak{M}$. The equation (4.7) for the components of the vector λ which are perpendicular to the field direction determines the spectrum of the transverse spin waves. Thus, for the circular components $\lambda_\pm=\lambda_x\pm i\lambda_y$ we have

$$\lambda_-(\omega-\mathbf{kv})-\zeta\left(\mathbf{kv}\delta_++\frac{1}{\hbar}\rho\mathfrak{M}\right)\int\lambda_-\,d\Gamma-\frac{2BH}{\hbar}\lambda_-=0. \tag{4.8}$$

The equation λ_+ can be obtained from (4.8) by making the formal substitutions $\omega\to-\omega$ and $\mathbf{k}\to-\mathbf{k}$. The integral equation (4.8) is equivalent to the following dispersion equation that determines the spectrum of the transverse spin waves:

$$1+\frac{ap_0}{\pi\hbar}\left\{w_+(u)+w_-(u)+\frac{p_0}{\hbar k}\left[-\frac{u^2-\chi_+^2}{u}w_+(u)+\frac{u^2-\chi_-^2}{u}w_-(u)+\frac{2}{u}\frac{N_+-N_-}{N_3}\right]\right\}=0, \tag{4.9}$$

where

$$w_\pm(u)=\frac{u}{2}\ln\frac{u+\chi_\pm}{u-\chi_\pm}-\chi_\pm,\qquad u=\frac{\omega-2BH/\hbar}{v_0k},\qquad v_0=\frac{p_0}{M},$$

with $BH=\beta H-\zeta(N_+-N_-)$. The condition that there be no Landau damping in this case takes the form $|u|>\chi_+$. In the region of small wave vectors $u\gg\chi_+$, the energy spectrum (4.9) is quadratic in k:

$$\omega=\frac{2\beta H}{\hbar}+\frac{\hbar k^2}{2M}\frac{(36\pi)^{1/3}}{10|a|}\frac{(N_+^{5/3}-N_-^{5/3})}{(N_+-N_-)^2} \tag{4.10}$$

and is similar, accurate to terms that are small in the concentration, to the spectrum obtained by Abrikosov and Dzyaloshinskiĭ,[10] for a ferromagnetic Fermi liquid in the absence of a field, while in weak fields $\mathcal{H}\ll1$ it goes over into the corresponding result of Silin.[15]

In the short-wave region $kv\gg\zeta(N_+-N_-)/\hbar$, the solution of (4.9) is exponentially close, in terms of the parameter $-\pi\hbar/p_+|a|\varkappa$, to the asymptotic solution: $\omega-2BH/\hbar=kv_+$. The region of existence of this solution is bounded with respect to frequency and to the wave vector of the magnons by the condition of applicability of the quasiclassical kinetic equation (4.1).

B. Spin-sound oscillations

Transverse spin waves, as shown above, are not connected with the oscillations of the superfluid background. Equation (4.7) for the z-component of the vector λ and (4.6) determine the propagation velocities of the coupled spin wave and the high-frequency sound wave. We note that in second-order of perturbation theory (see the Appendix) the coupling of the spin and sound modes in a magnetic field is caused also by the presence of a term linear in the spin operators $\varphi(\sigma_{\alpha\beta}\delta_{\mu\nu}+\sigma_{\mu\nu}\delta_{\alpha\beta})$ in the Fermi-liquid function.

We confine ourselves to the approximation linear in the concentration. To this end it suffices to retain in the energy ϵ_0 only the term that does not depend on the momentum: $\epsilon_0\approx\Delta$. With this same accuracy we have $\partial B/\partial N_4=0$. We solve Eqs. (4.2) and (4.3) with respect to $\mathbf{v}_s$ and δN_4 substitute the obtained expressions in (4.6) and (4.7), which are then reduced by the standard procedure to an algebraic system of linear equations:

$$\begin{gathered}[1-(\psi+Q)\Omega_+]x_1-R\Omega_+x_2-\zeta\Omega_-x_3=0,\\-(\psi+Q)Ms\Omega_+x_1+(1-MsR\Omega_+)x_2-Ms\zeta\Omega_-x_3=0,\\-(\psi+Q)\Omega_-x_1-R\Omega_-x_2+(1-\zeta\Omega_+)x_3=0,\end{gathered}$$

where

$$x_1=\int \mathrm{v}\,d\Gamma,\quad x_2=\int (\mathbf{k}/k)\mathbf{p}\mathrm{v}\,d\Gamma,\quad x_3=\int \lambda_z\,d\Gamma,$$

$$\Omega_\pm(u)=\frac{3N_3}{2p_0v_0}[w_+(u)\pm w_-(u)],\quad u=\frac{\omega}{kv_0}=\frac{s}{v_0},$$

$$Q(s)=\frac{1}{m_4(s^2-s_4{}^2)}\left[\frac{\rho_s}{m_4}\left(\frac{\partial\Delta}{\partial N_4}\right)^2-s^2\delta m\frac{m_3}{m_4}\frac{\partial\mu_4}{\partial N_4}-s^2\frac{\partial(-\Delta)}{\partial N_4}(2m_3-M)\right],$$

$$R(s)=\frac{s}{m_4(s^2-s_4{}^2)}\left[\frac{\partial(-\Delta)}{\partial N_4}+\frac{\delta m}{m_4}\frac{\partial\mu_4}{\partial N_3}\right];$$

$s_4^2=(\rho_s/m_4^2)\partial\mu_4/\partial N_4$ is the square of the velocity of fourth sound in the solution, and ρ_s is the density of the superfluid component. Calculating the determinant of this system, we obtain a transcendental dispersion equation for the propagation velocity of the coupled spin-sound oscillations:

$$1-{}^1\!/_2\Phi(w_++w_-)+Z(\Phi-Z)w_+w_-=0. \qquad (4.11)$$

In Eq. (4.11) we have

$$\Phi(s)=3\frac{N_3}{p_0v_0}[Q(s)+MsR(s)],\quad Z=3\frac{N_3}{p_0v_0}\zeta$$

and we recognize that (3.3) yields $\psi=-\zeta$. The dispersion equation (4.11) always has a root of the order of the speed of sound s_0 in pure He^4 and this determines the propagation velocity s_1 of the high-frequency sound wave in the solution. At $s\sim s_0\gg v_+$ we have $w_\pm\approx\chi_\pm{}^3v_0^2/3s^2$ and in the approximation linear in the concentration the propagation velocity of the high-frequency first sound is determined by the equation

$$1-\Phi(s_1)p_0v_0/3Ms_1{}^2=0.$$

Substituting the chemical potential μ_4 [Eq. (3.9)] in the expressions for the functions $Q(s)$ and $R(s)$ we obtain ultimately (cf.[9,16]):

$$s_1{}^2=s_0{}^2+s_0{}^2\frac{N_3}{N_4}\left[\frac{m_4}{M}\left(\alpha+\frac{\delta m}{m_4}\right)^2+\Lambda-\frac{\delta m}{m_4}\right]\approx s_0{}^2\left(1-0.29\frac{N_3}{N_4}\right),$$

where $\Lambda=(N_4^2/m_4s_2^0)\cdot\partial^2(-\Delta)/\partial N_4^2\approx 2.26$. This mode constitutes oscillations of the density of the impurity atoms He^3 and the associated oscillations of the projection of the magnetic moment on the direction of the external field. The dependence of the wave-propragation velocity s_1 in the magnetic field arises in higher orders in the fermion concentration.

It is easy to verify that in strong magnetic fields, when $v_+\gg v_-$, Eq. (4.11) has no real root different from $s=s_1$. On the other hand, in very weak magnetic fields

$$(v_+-v_-)/v_0\sim\beta H/T_0\ll\exp(-2/Z)\ll 1,\ T_0=p_0{}^2/2M\ .$$

Eq. (4.11) certainly has a solution close to the propagation velocity of the spin waves in the solution in the absence of the field S_0, given with logarithmic accuracy by $S_0=v_0[1+e^{-2/Z}]$. In this case Eq. (4.11) can be expanded in powers of the small parameter $y=\beta He^{-2/Z}/4T_0$ and takes the form

$$\left(w_0-\frac{1}{Z}\right)\left(w_0-\frac{1}{\Phi-Z}\right)=\left[\frac{1}{2}\frac{\beta H}{T_0}\frac{1}{u^2-1}\cdot\frac{1}{1+Z}\right]^2, \qquad (4.12)$$

where the function w_0 is determined by the expression for $w_\pm$ from (4.9) at $\chi_\pm=1$, $u=s/v_0$. Equation (4.12) determines the dependence of the velocity of the longitudinal spin wave S_H on the magnetic field:

$$S_H=S_0-v_0\frac{Z}{\Delta_s}y^2e^{-2/Z},\quad \Delta_s=\frac{2\zeta N_4+m_4(s_0\alpha)^2}{\zeta N_4+m_4(s_0\alpha)^2}\approx 1.03.$$

With increasing magnetic field, when a certain critical value H_c is reached, the propagation velocity of the spin wave becomes complex (the real part of the velocity is smaller here than the Fermi velocity v_+, and the appearing imaginary part turns out to be of the same order as the real part), and the wave experiences a strong Landau damping due to decay of the magnon into a particle and a hole.

The region of the existence of the longitudinal spin waves is bounded, according to (4.11) by the condition

$$w_+=\frac{\Phi w_--2}{2Z(\Phi-Z)w_--\Phi}>0. \qquad (4.13)$$

At $S_H\sim v_+$ we have

$$\Phi(s)\approx-\frac{m_4}{N_4}(s_0\alpha)^2\frac{3N_3}{p_0v_0}\equiv\Phi_0<0.$$

Therefore the condition (4.13) is equivalent to

$$w_-(u)>\frac{\Phi_0}{2Z(\Phi_0-Z)}. \qquad (4.14)$$

Reaching the critical field H_c is equivalent to obtaining equality in (4.14). Near the critical field, when the denominator of (4.13) vanishes, the spin-wave velocity satisfies the inequality $S_H-v_+\ll v_+-v_-$, and it is seen from (4.14) that in such fields the quantity $(v_+-v_-)/v_0$ is still exponentially small

$$\frac{v_+-v_-}{v_0}<\exp\left[-\frac{\Phi_0}{Z(\Phi_0-Z)}\right]\ll 1.$$

From this we determine, with logarithmic accuracy, the critical field

$$\beta H_c\approx T_0\exp[-\Phi_0/Z(\Phi_0-Z)],$$

corresponding to the numerical equation

$$H_c\,[\mathrm{Oe}]\approx 10^7x^{2/3}\exp(-1.16/x^{1/3}),$$

where x is the molar concentration of the He^3.

5. MAGNETIC KINETIC PHENOMENA

With increasing magnetic field, the mean free path of the quasi-particles with spins directed along **H** increases, inasmuch as the effective scattering for such particles is from particles with spins directed counter to the field, whose number decreases with increasing **H**. The corresponding magnetokinetic effects manifest themselves particularly noticeably in the determination of the kinetic coefficients, whose values are proportional to the particle mean free path.

A. Partially polarized solution

To determine the kinetic coefficients it is necessary to solve the collision kinetic equation for the density matrix. In the left-hand side of the kinetic equation (4.1), as always in the case of weakly inhomogeneous systems, it is necessary to substitute the local-equilibrium density matrix and the equation must be linearized in the small values of the gradients. If the z axis is chosen along the magnetic field, the equilibrium density matrix and the Hamiltonian of the excitations (2.1)–(2.3) and (2.12) are diagonal in spin space. We therefore obtain in place of four equations for $n_{\alpha\beta}$ two equations for the diagonal components of the density matrix:

$$\frac{\partial n^\pm}{\partial t}+\nabla n^\pm\frac{\partial \varepsilon_\pm}{\partial \mathbf{p}}-\frac{\partial n^\pm}{\partial \mathbf{p}}\nabla\varepsilon_\pm=I^\pm(\mathbf{p}). \qquad (5.1)$$

The diagonal elements of the density matrix $n^\pm$ and the excitation energies $\epsilon_\pm$ determine the distribution function and the energy of particles with definite spin projections on the field direction. In the principal approximation in the concentration, the collision integrals in (5.1) take the usual form:

$$\begin{aligned}I^\pm(\mathbf{p}_1)=-\int w\{&n_1{}^\pm n_2{}^\mp(1-n_{1'}{}^\pm)(1-n_{2'}{}^\mp)\\&-n_{1'}{}^\pm n_{2'}{}^\mp(1-n_1{}^\pm)(1-n_2{}^\mp)\}\\&\times\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1{}'-\mathbf{p}_2{}')\\&\times\delta(\varepsilon_1+\varepsilon_2-\varepsilon_1{}'-\varepsilon_2{}')\,d^3p_2\,d\Gamma_1{}'\,d\Gamma_2{}',\end{aligned} \qquad (5.2)$$

where $w=(2\pi\hbar)^3(a/M)^2$ is the probability of scattering of particles with momenta $\mathbf{p}_1^\pm$ and $\mathbf{p}_2^\mp$ into the states $\mathbf{p}_{1'}^\pm$ and $\mathbf{p}_{2'}^\mp$.

The procedure of determining the kinetic coefficients differs from the standard procedures in that the single kinetic equation for the particles on the Fermi surface $|\mathbf{p}|=p_0$ is replaced by two kinetic equations (5.1) that are connected via the collision integrals (5.2) for distribution functions of the $n^\pm$ particles situated in Fermi spheres having different radii $p_\pm$. The connection between Eqs. (5.1) for the occupation numbers n^+ and n^- means that in the kinetic processes there are, generally speaking, four relaxation times and the effects of dragging of particles with spin different spin orientations can become substantial. In first-order approximation, however, when the scattering probability does not depend on the angles, there are no dragging effects.

After cumbersome calculations analogous to those given in[7,17] we obtain for the viscosity and thermal-conductivity coefficients the expressions

$$\frac{\eta(\mathscr{H})}{\eta(0)}=\frac{\chi_+{}^5}{2d^3}\frac{2+5d^8-3d^{10}}{5-3d^2},\qquad d=\frac{\chi_-}{\chi_+},$$

$$\frac{\varkappa(\mathscr{H})}{\varkappa(0)}=\frac{\chi_+{}^3}{2d}\frac{4+3d^4+d^6}{3+d^2}$$

$$\varkappa(0)=\frac{1}{8\pi}\frac{p_0{}^3}{(Ma)^2}\frac{1}{T},\qquad \eta(0)=\frac{1}{12\pi^3}\frac{p_0{}^5}{(MaT)^2}. \qquad (5.3)$$

Figure 2 shows plots of the functions $\eta(\mathscr{H})/\eta(0)$ and $\varkappa(\mathscr{H})/\varkappa(0)$, obtained from formulas (5.3). In strong magnetic fields $\mathscr{H}\to 1$ the viscosity and thermal-conductivity coefficients (5.3) increase without limit:

$$\frac{\eta(\mathscr{H})}{\eta(0)}=\frac{2^{5/3}}{5(1-\mathscr{H})^{3/2}},\qquad \frac{\varkappa(\mathscr{H})}{\varkappa(0)}=\frac{4}{3(1-\mathscr{H})^{1/2}}.$$

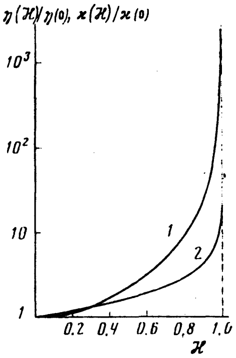

FIG. 2. Relative change of the kinetic coefficients as a function of the magnetic field: 1—visocity $\eta(\mathscr{H})/\eta(0)$, 2—thermal conductivity $\varkappa(\mathscr{H})/\varkappa(0)$.

B. Fully polarized solutions

The limiting values of the kinetic coefficients are reached in a fully polarized solution at $\mathscr{H}\geq 1$. In this case only one of the equations in (5.1) is left, and to determine the kinetic coefficients we can use the exact expression[18]:

$$\eta T^2=\frac{64}{45}\frac{\hbar^3p_F{}^5}{m_+{}^4}\left\langle\frac{w(\theta,\varphi)}{\cos(\theta/2)}(1-\cos\theta)^2\sin^2\varphi\right\rangle^{-1}C_\eta,$$

$$\varkappa T=\frac{8\pi^2}{3}\frac{(\hbar p_F)^3}{m_+{}^4}\left\langle\frac{w(\theta,\varphi)}{\cos(\theta/2)}(1-\cos\theta)\right\rangle^{-1}H_\varkappa,$$

where $w(\theta,\varphi)$ is the particle collision probability and is connected with the amplitude of the p-scattering (3.10) (cf.[8]):

$$w(\theta,\varphi)=\frac{\pi}{2\hbar}|\Gamma(\mathbf{p}_F,\mathbf{p}'_F)|^2=\frac{\pi}{2\hbar}\left(\frac{24\pi b}{M}\right)^2p_F{}^4\sin^4\frac{\theta}{2}\cos^2\varphi, \qquad (5.4)$$

$\langle\cdots\rangle$ denotes averaging over the angles, and the coefficients C_η and $H_\varkappa$[18] for the function w (5.4) turn out in this case to be equal to $C_\eta=0.79$. $H_\varkappa=0.55$. Ultimately we get

$$\eta T^2=\frac{7}{\pi}\left(\frac{\hbar^2}{24\pi bM}\right)^2p_F\cdot 0.79,\qquad \varkappa T=\frac{35\pi}{6}\left(\frac{\hbar^2}{24\pi bM}\right)^2\frac{1}{p_F}0.55, \qquad (5.5)$$

i.e., their concentration dependences are significantly altered and they differ from their values $\eta(0)$ and $\varkappa(0)$ [Eq. (5.3)] in the absence of a field by a large factor of the order of

$$(a^6/b^2)(N_3a^3)^{-4/3}\gg 1.$$

6. CONCLUSION

The influence of the magnetic field on the greater part of the considered phenomena is nelgigible in weak fields (at $\mathscr{H}\ll 1$ the contribution of the magnetic field is proportional to $\mathscr{H}^2$), becomes substantial only at a high degree of polarization of the solution. Polarization of the solution leads to an effective weakening of the interaction by a factor $x^{-4/3}$ ($x\ll 1$ is the fermion concentration), since the s-scattering and p-scattering amplitudes of slow particles differ by the large factor $x^{-2/3}$. The interaction is due to the p-scattering only when the ratio of the number of particles with spins antiparallel and parallel to the field is less than $x^{4/3}\ll 1$. For degenerate systems $T \ll T_F$ this is equivalent to the following condition on the field ($\mathscr{H} > 1$)

$$1 \gg x^{1/3} \gg \exp\{(1-\mathscr{H}) T_F/T\}.$$

At sufficiently low temperatures, the conditions of full polarization of the solution $\mathscr{H} \sim 1$ corresponds to the numerical equality $H[\mathrm{kOe}] = 2.6 \times 10^4\, x^{2/3}$. At a concentration $x \sim 10^{-4}$ the field is $H \approx 50$ kOe, and the degeneracy temperature is $T_F \approx 8$ mK. The kinetic coefficients (5.5) then exceed their values in the absence of a field by a factor $x^{4/3} \approx 10^5$. Thus, the viscosity at $x \sim 10^{-4}$ increase as a result of the magnetization from the value $\eta(0) \sim 10^7 (T_F/T)^2$ [poise] to the value $\eta(\mathscr{H} > 1) \sim 10^{-2} (T_F/T)^2$ [poise]. Thus, the increase of the field leads to a unique magnetic effect–an appreciable increase of the particle mean free path l and of the relaxation time τ. In the absence of a field $l(\mathscr{H}=0) \sim (a^2 N_3)^{-1} (T_F/T)^2 \sim (a/x)(T_F/T)^2$. In a polarized solution the mean free path $l(\mathscr{H} > 1) \sim l(0)/x^{4/3} \sim (a/x^{7/3})(T_F/T)^2$ reaches a value $l \sim 10 (T_F/T)^2$ cm ($x \sim 10^{-4}$), and consequently a Knudsen regime is realized by the fermions and various size effects can be observed.

So appreciable an increase of the relaxation time means that the region of existance of the weakly damped high-frequency modes $\omega\tau \gg 1$ is noticeably increased, a fact corresponding to the condition $\omega \gg 5(10x^{1/3})^4 T^2$, where T is the temperature in mK. By virtue of the inverse condition, the undamped hydrodynamic oscillations of a strongly polarized solution are in fact quasistatic.

The employed approach is valid also in the case of finite temperatures, $T \neq 0$, for nondegenerate low-density system (it is known that in first-order perturbation theory the Fermi quasiparticles are undamped at any ratio of T and T_F). Thus, the transition to the case of a completely polarized solution goes through a system state wherein the subsystem of the particles with spins parallel to the field is degenerate, and the number of particles with spins oriented against the field is small, and these particles have a Boltzmann distribution.

Many of the results can be used also for other types of Fermi-liquid of low density, in which the interaction between the bare particles decreases rapidly enough with increasing distance. This condition is satisfied for electrons when the interaction is determined by a screened Coulomb potential. For charged fermions, however, the Lorentz force plays a major role. Therefore the results of this paper are directly applicable to electrons when the non-equilibrium population of the spin states is not due to the presence of an external magnetic field and arises, for example, as a result of injection of spins having a definite orientation.

An example of a completely polarized Fermi liquid of low density is provided by the vacancions in the ferromagnetic phase in crystalline He^3.[19] The fermion concentration in such a phase is $x \sim 6 \times 10^{-3}$, and the Fermi energy is $T_F \sim 0.3$ K. The quasiparticle mean free path is determined by the collisions of the vacancions with one another and at $a \sim 10^{-8}$ cm it turns out to be quite appreciable–of the order of $l \sim 100/T^2$ cm (T is the temperature in mK).

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APPENDIX

A number of Fermi-liquid effects do not arise in first-order perturbation theory. For example, the effective mass (2.14) differs from the bare mass in second order in the interaction. The second-order correction to the ground-state energy per unit volume (3.2) is equal to[2-4]

$$-\frac{16\pi^2 a^2 \hbar^4}{M} \sum_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4} \frac{n^{(0)}_{\gamma\varepsilon}(\mathbf{p}_1)\, n^{(0)}_{\zeta\eta}(\mathbf{p}_2)\, n^{(0)}_{\varkappa\varkappa}(\mathbf{p}_3)}{p_1^2+p_2^2-p_3^2-p_4^2} (\delta_{\varepsilon\gamma}\delta_{\eta\zeta} - \sigma_{\varepsilon\gamma}\sigma_{\eta\zeta}). \qquad \text{(A.1)}$$

To determine the Fermi-liquid function $f_{\alpha\beta,\,\mu\nu}(\mathbf{p}, \mathbf{p}')$ in second-order perturbation theory we vary (A.1) with respect to $n^{(0)}_{\beta\alpha}(\mathbf{p})$ and $n^{(0)}_{\nu\mu}(\mathbf{p}')$:

$$f^{(2)}_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}') = -\frac{32\pi^2 a^2 \hbar^4}{M} \sum_{\mathbf{p}_1,\mathbf{p}_2} \left\{ \frac{n(\mathbf{p}_1)(\delta_{\alpha\beta}\delta_{\mu\nu} - \sigma_{\alpha\beta}\sigma_{\mu\nu})}{p^2+p'^2-p_1^2-p_2^2} + \frac{n(\mathbf{p}_1)\delta_{\alpha\beta} - \rho(\mathbf{p}_1)\sigma_{\alpha\beta}}{p^2+p_1^2-p'^2-p_2^2}\delta_{\mu\nu} + \frac{n(\mathbf{p}_1)\delta_{\mu\nu} - \rho(\mathbf{p}_1)\sigma_{\mu\nu}}{p'^2+p_1^2-p^2-p_2^2}\delta_{\alpha\beta} \right\}. \qquad \text{(A.2)}$$

As a result of integration, the f-function (A.2) takes the form (2.5), where

$$\begin{gathered} \psi^{(2)}(\mathbf{p},\mathbf{p}') = -C(I_1^+ + I_1^- + I_2^+ + I_2^- + I_3^+ + I_3^-), \\ \zeta^{(2)}(\mathbf{p},\mathbf{p}') = C(I_1^+ + I_1^-), \quad \xi(\mathbf{p},\mathbf{p}') = 0, \\ \varphi(\mathbf{p},\mathbf{p}') = C(I_2^+ - I_2^-), \quad \varphi(\mathbf{p}',\mathbf{p}) = C(I_3^+ - I_3^-) \end{gathered} \qquad \text{(A.3)}$$

and the following notation has been introduced:

$$\begin{gathered} I_1^{\pm} = \frac{q}{4}\ln\frac{p_\pm q + p_\pm^2 - \mathbf{p}\mathbf{p}'}{p_\pm q - p_\pm^2 + \mathbf{p}\mathbf{p}'} - p_\pm - \frac{2p_\pm^2 - p^2 - p'^2}{4r}\ln\frac{p_\pm r + p_\pm^2 + \mathbf{p}\mathbf{p}'}{p_\pm r - p_\pm^2 - \mathbf{p}\mathbf{p}'} \\ I_2^{\pm} = \frac{p_\pm}{q^2}(\mathbf{p}\mathbf{p}' - p'^2) - \frac{q^2 p_\pm^2 - (\mathbf{p}\mathbf{p}' - p'^2)^2}{2q^3}\ln\frac{p_\pm q + p'^2 - \mathbf{p}\mathbf{p}'}{p_\pm q - p'^2 + \mathbf{p}\mathbf{p}'}, \\ I_3^{\pm}(\mathbf{p},\mathbf{p}') = I_2^{\pm}(\mathbf{p}',\mathbf{p}), \quad C = \frac{2a^2\hbar}{M}, \quad r = |\mathbf{p}+\mathbf{p}'|, \quad q = |\mathbf{p}-\mathbf{p}'|, \end{gathered} \qquad \text{(A.4)}$$

and for $p_\pm$ we can use their ideal-gas values $\chi_\pm p_0$(3.7). At $H=0$ expressions (A.3) and (A.4) coincide with those given in[4]. In concrete calculations, the only important values of the Fermi-liquid function are those on Fermi surfaces with radii p_+ and p_-.

In the presence of a Bose background, the next-order correction cannot be obtained in similar fashion, since the correction to the f-function due to the effective retardation, $f^{(s)}_{\mathrm{ret}} \propto (p_0/p_c)^2$, is of the same order of smallness as the third-order perturbation theory term in s-scattering and the principal term corresponding to p-scattering (3.13). For this reason, the p scattering for fermions with a Bose background can be taken into account only for systems with sufficiently high degree of polarization.

The contribution of the retardation to p scattering, $f^{(p)}_{\mathrm{ret}} \propto (p_F/p_c)^4$, greatly exceeds the second-order corection $f^{(p)}_{\mathrm{ret}} \propto (p_F/p_c)^5$ to (3.13).

[1] For the same reason, the osmotic pressure increases in strong magnetic fields by a factor $2^{2/3}$.

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Phonon spectrum and local vibrations in strongly anisotropic crystals

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The polarization of vibrations and behavior of the vibration plane in layer and chain crystals are investigated throughout the continuous spectral band. It is shown that there are at least four critical points. The density of flexural vibrations exhibits a standard square-root kink only is very narrow regions next to critical points located close to the lower limit of the spectrum. Outside these regions the behavior of the vibration density is quite different. The influence of anisotropy on the properties of local vibrations is considered. It is established that the threshold of appearance of local vibrations is considerably less than in an isotropic crystal and that the shape of the localization region is extremely anisotropic.

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1. INTRODUCTION

In 1952, I.M. Lifshitz[1] first drew attention to the existence—in strongly anisotropic crystals with layer or chain structures—of "flexural" vibrations resembling flexural waves in noninteracting atomic layers or chains. He pointed out a considerable qualitative difference between long-wavelength flexural vibrations and vibrations polarized in the plane of a layer or along a chain. This difference is manifested in a number of properties of a crystal and, in particular, it gives rise to an unusual temperature dependence of the specific heat at low temperatures.

The recent years have seen concentration of attention on the conducting properties of strongly anisotropic crystals. Less work has been done on the phonon spectra of these crystals. This accounts for the absence, up until now, of a complete description of the properties of the density of vibrations in such crystals. Only some characteristic features of the behavior of this density are known for layer crystals at low frequencies[2] and these indicate that the influence of anisotropy on the phonon spectrum may be very considerable. For example, the formation of a sharp peak on the curve representing the density of flexural vibrations near the lower limit of the spectrum is reported by Kosevich.[2] However, the parameters of this peak have not yet been investigated. Moreover, the problem of the number, nature, and positions of the Van Hove critical points throughout the vibration spectrum of these crystals has not been tackled.

There is also considerable interest in the properties of local vibrations in strongly anisotropic crystals. The existence of local vibrations near point defects in crystals was first pointed out by I. M. Lifshitz in 1947.[3] These vibrations appear near various imperfections of the lattice and have been investigated quite thoroughly for isotropic crystals.[4,5] However, up to now the problem of the influence of a strong anisotropy on the properties of local vibrations has not been considered. We may expect local vibrations in strongly anisotropic crystals to have properties different from those in isotropic crystals. In fact, since planar layers and linear chains of atoms with a strong interaction are physically