Suppression of transverse zero sound in He^3 by a magnetic field

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The conditions for the existence of undamped high-frequency oscillations in normal He^3 in a magnetic field are determined. The critical field in which a transverse zero-sound wave is suppressed is found by calculation to be $H_c \approx 4$ kOe.

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The influence of the magnetic field H on the properties of superfluid He^3 in realistically attainable fields is negligible to the extent of the smallness of the ratio $\beta H/T_F$(β is the magnetic moment of the He^3 atom, T_F is the degeneracy tempera-

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ture). However, the condition for the existence of weakly damped zero-sound and spin oscillations $\omega > kv_0$[1] (ω and **k** are the frequency and the wave vector, v_0 is the Fermi velocity in the absence of a magnetic field) may turn out to be sensitive to the field even in weak fields, if, for example, the propagation velocity of the oscillations is close to the Fermi velocity. Thus,[2] in the solution He^3–HeII, the condition for the propagation of a longitudinal spin wave, $\omega > kv_+$(v_+ and v_- are the velocities on the Fermi surfaces for quasiparticles polarized parallel and antiparallel to the field) is violated in a weak field $\beta H/T_F \ll 1$, and this leads to a strong Landau damping connected with the decay of the magnon into a quasiparticle and a hole.

In the present study we investigated the conditions for the appearance and vanishing of zero-sound and spin waves in normal He^3 in a magnetic field. The system of equations describing the high-frequency oscillations of a Fermi liquid in a magnetic field consists of the thoroughly studied[3,4] equation of motion for the magnetic-moment component perpendicular to **H** and of two coupled equations for the longitudinal magnetization and the scalar distribution function. We are interested in the last two equations in the approximation linear in **H** (cf.[2]):

$$(\omega - \mathbf{kv})\lambda - \delta_+ \mathbf{kv}\left(\int \zeta\lambda' d\Gamma' + \int H\phi\nu' d\Gamma'\right) - \delta_- \mathbf{kv}\int \psi\nu' d\Gamma' = 0,$$
$$(\omega - \mathbf{kv})\nu - \delta_+ \mathbf{kv}\left(\int \psi\nu' d\Gamma' + \int H\phi\lambda' d\Gamma'\right) - \delta_- \mathbf{kv}\int \zeta\lambda' d\Gamma' = 0. \qquad (1)$$

Here $\nu + \lambda\boldsymbol{\sigma}$ ($\boldsymbol{\sigma}$ are Pauli matrices) is the nonequilibrium increment to the single-particle density matrix, $\lambda \equiv \lambda_z$, **v** is the quasiparticle velocity, $\delta_\pm = \frac{1}{2}[\delta(\epsilon_+ - \mu) \pm \delta(\epsilon_- - \mu)]$, $\epsilon_\pm$ is the Fermi energy of quasiparticles with different spin orientations, μ is the chemical potential, and $d\Gamma = 2d^3p/(2\pi\hbar)^3$. Equation (1) makes allowance for the fact that in weak fields, accurate to terms quadratic in **H**, the Fermi-liquid function is given by

$$f_{\vec{\sigma}\vec{\sigma}'}(\mathbf{p},\mathbf{p}') = \psi(\theta) + \zeta(\theta)\vec{\sigma}\vec{\sigma}' + \phi(\theta)(\vec{\sigma} + \vec{\sigma}')\mathbf{H}, \qquad (2)$$

where θ is the angle between the vectors **p** and **p**′, while the functions ψ, ζ, and ϕ are determined by their values in the absence of a field. With the same accuracy we have $\delta_+ = \delta(\epsilon_0 - \mu)$, and δ_- is linear in **H**. By expanding the f-function in Legendre polynomials we reduce the equations in (1) in standard fashion[5] to a system of linear equations for the quantities ν_{nm} and λ_{nm}

$$\nu_{km} = \frac{1}{2}F_k^+ \sum_n \Omega_{nk}^{m+}\left(\nu_{nm} + \lambda_{nm} + H\frac{\Phi_k}{Z_k}\lambda_{nm}\right) + \frac{1}{2}F_k^- \sum_n \Omega_{nk}^{m-}\left(\nu_{nm} - \lambda_{nm} + H\frac{\Phi_k}{Z_k}\lambda_{nm}\right)$$
$$\lambda_{km} = \frac{1}{2}Z_k^+ \sum_n \Omega_{nk}^{m+}\left(\lambda_{nm} + \nu_{nm} + H\frac{\Phi_k}{F_k}\nu_{nm}\right) + \frac{1}{2}Z_k^- \sum_n \Omega_{nk}^{m-}\left(\lambda_{nm} - \nu_{nm} + H\frac{\Phi_k}{F_k}\nu_{nm}\right). \qquad (3)$$

Here $|m| \leq k, n$; $F_k^\pm = (v_\pm/v_0)F_k$, $Z_k^\pm = (v_\pm/v_0)Z_k$, and F_k, Z_k, Φ_k are the usual harmonics of the functions ψ, ζ, ϕ (2) in the absence of a field. The function Ω in (3) is defined by

$$\Omega^{\pm}(s_{\pm}) = \Omega\left(\frac{\omega}{kv_{\pm}}\right);\ \Omega^m_{kn}(s) = \frac{(k-|m|)!}{(k+|m|)!}\int_{-1}^{1}\frac{dx}{2}\frac{x}{s-x}P^m_n(x)\,P^m_k(x)$$

$P^m_n(x)$ are associated Legendre polynomials.

The propagation velocity of the high-frequency oscillations of He^3 in a magnetic field $u(\mathbf{H})$ is determined from the condition that the determinant of the system (3) vanish: $D(s_+,s_-)=0$. The vanishing or the appearance of some mode occurs in such a field H_c near which the solution of this dispersion equation takes the form $s_+\to 1$, $s_-\to v_+/v_-\equiv 1+h_c$, with $h_c=(\beta H_c/T_F)/(1+Z_0)\ll 1$. The dispersion equation $D(1,1+h_c)=0$ determines in fact the critical magnetic field H_c.

Since the harmonics $F_k, Z_k, \Phi_{k,k} \geqslant 2$ are presently unknown, it cannot be definitely concluded that solutions of Eqs. (3) exist for oscillations with azimuthal number $m\geqslant 2$.

The case $m=0$ is of no interest, since the velocity of the longitudinal zero sound is much larger than the Fermi velocity in the absence of a field.

Of greatest interest is the recently observed transverse ($m=1$) zero sound,[6,7] whose propagation velocity u_1 in the absence of a field is apparently close to the Fermi velocity $(u_1-v_0)/v_0\equiv a_0\ll 1$. The function Ω^1 has a logarithmic singularity in the first derivative as $s\to 1$. Neglecting the terms linear in the field of order $h\equiv(\beta H/T_F)/(1+Z_0)\ll 1$ in comparison with terms of order $h\ln h$, we must put $F^{\pm}_k=F_k$, $Z^{\pm}_k=Z_k$, and $H\Phi_k=0$, in (3), and in the difference $\Omega^{1\pm}(s_{\pm})-\Omega^1(1)$ it suffices to retain the principal term of order $h\ln h$ (account is taken of the fact that $(v_{\pm}-v_0)/v_0=\pm h/2$). The dispersion equation takes the form

$$D(s_+, s_-) = D(1, 1) + A\,(a^+\ln a^+ + a^-\ln a^-) = 0,$$

where A is a certain constant, and the quantities $\alpha^{\pm}(h)\ll 1$ determine the difference between the propagation velocity $u_1(H)$ of zero sound in the field and $v_{\pm}$:

$$a^{\pm}(h) \pm h/2 = [u_1(h) - v_0]/v_0,\quad a^{\pm} \equiv s_{\pm} - 1.$$

Inasmuch as in the absence of a field the dispersion equation is, with the same accuracy, of the form

$$D(1+a_0, 1+a_0) = D(1,1) + 2Aa_0\ln a_0 = 0,$$

it follows that the zero-sound velocity $u_1(h)$ in the field is easily expressed in terms its value $u_1(0)$ at $H=0$

$$2a_0\ln a_0 = a^+\ln a^+ + a^-\ln a^-, \tag{4}$$

and the critical field H_c, determined from the conditions $\alpha^+=0$ and $\alpha^-=h$, is specified by the equation

$$2a_0\ln a_0 = h_c\ln h_c. \tag{5}$$

Equations (4) and (5), the accuracy of which is characterized by the inequality

$$|\ln a_0| >> 1$$

take into account all the harmonics of the function F. The measurment of H_c is thus a method of directly determining the velocity of the transverse zero sound in the absence of a field $u_1(0)$, and can yield information on the higher-order harmonics of F. If, as usual, we confine ourselves only to F_1 and F_2,[8,9] then

$$a_0 \ln a_0 = \frac{F_1 - 6 + 3F_2/(1+F_2/5)}{3F_1 + 9F_2/(1+F_2/5)}$$

and it is possible to use (5) to determine F_2. For the critical field H_c at[10] $F_1=6.04$, $Z_0=-0.67$, $T_F=1.64$ K, and $F_2=0$ we obtain from (5) the estimate $H_c \approx 4$ kOe. The fact that the signal depends substantially on the magnetic field can help in the experimental separation of the contribution of the transverse zero sound, since a weak field has practically no effect on the propagation of ordinary Fermi quasiparticles in He^3.

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