Inhomogeneous phase of ^{3}He solution in superfluid ^{4}He

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The properties of Fermi systems with pairing with nonzero momentum are investigated with a ^{3}He solution in superfluid ^{4}He in a magnetic field as an example. Linearized equations of the hydrodynamics of the system near a second-order phase transition are obtained. They differ from the system of equations of three-velocity hydrodynamics of the superfluid homogeneous ^{3}He phase in a ^{3}He–He II solution in that they are essentially anisotropic and contain additional terms that characterize the elasticity of the structures. The spectrum of the possible long-wave hydrodynamic oscillations is determined. Additional modes of elastic rigid-body oscillations appear in the inhomogeneous phase. The dispersion law of the elastic and of the temperature oscillations is anisotropic.

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1. INTRODUCTION

The superfluid state of ^{3}He in a ^{3}He II solution differs considerably in its properties from the superfluid phases of pure ^{3}He, which are presently extensively investigated. Particularly distinguished is the phase transition of ^{3}He in the solution in the presence of an external magnetic field, where, depending on the field intensity, superfluid phases having entirely different properties are produced.[1]

In the ^{3}He–He II solution, the interaction of the ^{3}He quasiparticles, owing to the low ^{3}He density, reduces mainly to s-scattering, which has in a solution, in contrast to pure dense ^{3}He, the character of attraction.[2] Therefore the transition of ^{3}He in solution into the superfluid state in the absence of a magnetic field should be described within the framework of the usual BCS theory, which in the case of a solution provides not a model description of the transition, as for superconductors, but an exact one. The corresponding estimates of the temperature of the superfluid transition of ^{3}He in solution[3] seem quite promising and give grounds for hoping to observe a new type of superfluid system with two Bose condensates in the not too distant future.

In magnetic fields that are not too strong, the superfluidity of ^{3}He in solution is due, just as in the absence of a field, to s-pairing of the ^{3}He-quasiparticles. In a magnetic field, the Fermi momenta of ^{3}He quasiparticles with different spin orientations are not equal, and this hinders the formation of pairs with zero momentum. As a result, the temperature of the transition of the ^{3}He in solution into the superfluid state decreases with increasing field, and in a certain range of fields the formation of Cooper pairs with zero momentum is not favored. Pairs with nonzero momentum are then produced, and this leads to spatial inhomogeneity of the resultant low-temperature phase. In this range of fields, the superfluid solution acquires a periodic spatial structure. In still stronger fields, s-pairing of the fermions ^{3}He is not possible at all, and if the p-scattering of the ^{3}He quasiparticles in the solution, just as in pure ^{3}He, has the character of attraction, then the structure of the ^{3}He condensate in the solution with the corresponding phase is similar to the A-phase of superfluid pure ^{3}He in a strong (compared with the gap) magnetic field. Thus, an external magnetic field not only influences noticeably the properties of the normal ^{3}He phase dissolved in the He II,[4,5] but also alters radically the entire picture of the transition of the ^{3}He in solution into the superfluid state.

We report here an investigation of the properties of the spatially inhomogeneous phase of the solution. The thermodynamic properties of this phase are in many respects similar to the properties of the predicted inhomogeneous phase of superconductors.[6-9] Just as in superconductors,[7] a spin-wave density is produced in the inhomogeneous phase of the solution at a constant particle-number density. The velocity of the quasiparticles in a certain direction may turn out to be close to or equal to zero, and this leads to a strong anisotropy of the kinetic coefficients and to a slow decrease of the heat capacity with decreasing temperature.

However, there are also substantial differences between the properties of a solution and a superconductor, since the ^{3}He quasiparticles have no charge and the system contains two Bose condensates, ^{3}He and ^{4}He. For superconductors, the model of a spatially-inhomogeneous phase is quite crude, since the possibility of the existence of an inhomogeneous superconducting phase is strongly influenced by the electronic diamagnetism, by spin-orbit interaction, and by scattering from impurities. The same causes have not made it possible to observe so far a spatially inhomogeneous superconducting phase. No such difficulties are encountered for ^{3}He–^{4}He solutions. This makes possible a quantitative theoretical and experimental investigation of a condensate with a spatial structure.

In the next section we present the principal relations that describe the inhomogeneous phase of the solution. In the third section we investigate the superfluid motion in a spatially inhomogeneous phase. The fourth section is devoted to the elastic properties of the structure. In the last part of the paper we discuss low-frequency (hydrodynamic and elastic) oscillations in the inhomogeneous phase of the solution. We employ the previously developed[10,11] method of describing the properties of a solution without resorting to any model represen-

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tation concerning the structure of the interaction and, where possible, use the notation employed in the cited reviews. Some preliminary results were published earlier[1] (see also Ref. 11).

2. PRINCIPAL PROPERTIES OF INHOMOGENEOUS PHASE

We are interested in the region of very low temperatures, when the number of rotons and phonons is vanishingly small, and the normal component of the solution is a degenerate low-density Fermi gas of slow ^{3}He quasiparticles immersed in a superfluid Bose background of He II. Since the system is isotropic and the density of the ^{3}He is low, the BCS theory yields a rather exact and not a model-dependent, as in the case of metals, description of the phase transition in the system of the ^{3}He quasiparticles.[3,10,11] In an external magnetic field H, with the z axis in spin space chosen along the field direction, the anomalous temperature Green's function $\hat{F}_{\alpha\beta}$ and the energy gap $\Delta_{\alpha\beta}$ for the ^{3}He quasiparticles (α and β are spinor indices) have the following spin dependence:

$$\hat{F}_{\alpha\beta}=i\sigma_{\alpha\beta}{}^{y}F(\zeta_s,\mathbf{r}_1,\mathbf{r}_2),\quad \Delta_{\alpha\beta}=i\sigma_{\alpha\beta}{}^{y}\Delta(\mathbf{r})$$

[σ^y is the y component of the Pauli matrices, $\zeta_s=\pi T\times(2s+1)$ is the Matsubara frequency, T is the temperature], and the system of Gor'kov equations is diagonalized in spin space and assumes in the absence of superfluid motion the usual form (here and elsewhere $\hbar=1$):

$$\begin{aligned}&\left(i\zeta_s+\frac{1}{2M}\nabla^{(1)\,2}+\mu_3\pm\beta H\right)G_{12}^{\pm}+\Delta(\mathbf{r}_1)F_{12}=\delta(\mathbf{r}_1-\mathbf{r}_2),\\&\left(-i\zeta_s+\frac{1}{2M}\nabla^{(1)\,2}+\mu_3\mp\beta H\right)F_{12}-\Delta^{*}(\mathbf{r}_1)G_{12}^{\pm}=0,\\&\Delta(\mathbf{r})=gT\sum_{s=-\infty}^{\infty}F(\zeta_s,\mathbf{r},\mathbf{r}),\quad g=4\pi|a|/M,\end{aligned}\tag{1}$$

where $G_{12}^{\pm}=G^{\pm}(\zeta_s,\mathbf{r}_1,\mathbf{r}_2)$ are the temperature (Matsubara) Green's functions for particles with different spin orientations, μ_3 is the chemical potential of ^{3}He in the solution, $\beta=0.08$ mK/kOe is the nuclear magnetic moment of the ^{3}He, M is the effective mass of the ^{3}He impurity quasiparticles, a is the s-scattering length, and $\nabla^{(1)}\equiv\partial/\partial r_1$. According to the experimental data (see, e.g., Refs. 10 and 11), at zero pressure we have $a=-1.5$ Å and $M=2.3m_3$ (m_3 is the mass of the ^{3}He atom). The equation for the other anomalous Green's function $F(\zeta_s,\mathbf{r}_1,\mathbf{r}_2)$ is obtained by conjugation from the second equation of (1).

Near the phase transition, when the energy gap Δ is small, Eqs. (1) are solved by expansion in powers of Δ. The expression for $\hat{F}$ in the approximation linear in Δ is then the equation for Δ:

$$\Delta^{*}(\mathbf{r})=gT\sum_{s=-\infty}^{\infty}\int d^3\mathbf{r}_2 G_{12}^{\mp(0)}\Delta^{*}(\mathbf{r}_1)G_{21}^{\pm(0)},\tag{2}$$

where $G_{12}^{\pm(0)}=G^{\pm(0)}(\zeta_s,\mathbf{r}_1,\mathbf{r}_2)$ are the usual temperature Green's functions of an ideal Fermi gas for particles with different spin orientations, with Fourier components

$$G_{12}^{\pm(0)}=\int\frac{d^3p}{(2\pi)^3}G_{\mathbf{p}}^{\pm}e^{i\mathbf{p}(\mathbf{r}_1-\mathbf{r}_2)},\quad G_{\mathbf{p}}^{\pm}=\left[i\zeta_s-\frac{p^2}{2M}+\mu_3\pm\beta H\right]^{-1},$$

and $\tilde{G}^{(0)}\equiv G^{(0)}(-\zeta_s)$. As a result of a Fourier transformation, Eq. (2) is reduced to the form

$$\Delta_Q\Pi(Q)=0,\quad \Pi(Q)=-\frac{1}{g}+T\sum_{s=-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}G_{\mathbf{p}}^{\mp}G_{\mathbf{p}-\mathbf{Q}}^{\pm},\tag{3}$$

Δ_Q is the Fourier component of the order parameter $\Delta(\mathbf{r})$.

The appearance of a nontrivial solution of Eq. (3) [the vanishing of $\Pi(Q)$] means instability of the normal phase with respect to formation of cooper pairs with momentum $\mathbf{Q}$. In the absence of a magnetic field, the highest is the instability temperature T_{c0} for pairs with zero momentum. The expression for the temperature T_{c0} of the superfluid transition and the corresponding numerical estimates were analyzed in detail in Refs. 3, 10, and 11. With decreasing density of the ^{3}He in the solution, the value of T_{c0} decreases exponentially, with $T_{c0}\gtrsim 1$ mK in solutions with maximum concentrations.

In the presence of a magnetic field, pairing with nonzero momentum $\mathbf{Q}$ may turn out to be more profitable. The real instability of the solution is connected with production of pairs with a momentum $\mathbf{Q}$ for which the instability temperature, specified by Eq. (3), is a maximum. Thus, the temperature of the absolute instability of the normal phase T_{cH} and the momentum $\mathbf{Q}(H)$ of the produced pairs are given by the equations[8]

$$\Pi(\mathbf{Q})=0,\quad \partial\Pi/\partial\mathbf{Q}=0.\tag{4}$$

An analysis of Eqs. (4) has shown[1,8,11] that in weak fields

$$\beta H<1.06T_{c0}$$

the transition goes over into the usual BCS phase, and in the field range

$$1.06T_{c0}<\beta H<1.33T_{c0}$$

a spatially inhomogeneous structure is produced. The temperature of the absolute instability of the normal phase with respect to the onset of such a structure decreases with increasing field, from $T_{cH}=0.56T_{c0}$ at $\beta H=1.06T_{c0}$, to zero at $\beta H=1.33T_{c0}$ [plots of $T_{cH}(H)$ and $Q(H)$ can be found, e.g., in Refs. 1 and 11].

In the spatially inhomogeneous phase, the equilibrium energy gap Δ has an explicit dependence on the coordinates:

$$\Delta(\mathbf{r})=\sum_m\Delta_m e^{i\mathbf{Q}_m\mathbf{r}},\tag{5}$$

where all the vectors $\mathbf{Q}_m$ given by Eqs. (4) are equal in magnitude: $|\mathbf{Q}_m|=Q$. The characteristic scale of the inhomogeneity is $Q\sim 1/\xi_0$, where $\xi_0\sim v_F/T_{c0}$ is the coherence length, which exceeds substantially the average distance $a_0x^{1/3}$ between the ^{3}He particles [$v_F=p_F/M$ is the Fermi velocity, a_0 is the atomic dimension, $x\sim(p_Fa_0)^3$ is the density of ^{3}He in the solution].

Unfortunately, the exact form of the $\Delta(\mathbf{r})$ dependence (5) has not yet been established. Nor do we know the curve of the first-order transition from the inhomogeneous phase into the ordinary superfluid BCS phase. What was investigated in detail was the thermodynamics of an inhomogeneous phase with an order parameter (5) that contains only one harmonic[6,8]:

$$\Delta(\mathbf{r})=\Delta_0 e^{i\mathbf{Q}\mathbf{r}}. \tag{6}$$

More interesting, however, are the symmetrical phases $\Delta(\mathbf{r})=\Delta(-\mathbf{r})$, for which the expansion (5) takes the form

$$\Delta(\mathbf{r})=2\sum_m \Delta_m \cos \mathbf{Q}_m\mathbf{r}. \tag{7}$$

It is most probable,[7,9] that the inhomogeneous phase has a layered

$$\Delta(\mathbf{r})=2\Delta_0 \cos \mathbf{Q}\mathbf{r} \tag{8}$$

or a cubic

$$\Delta(\mathbf{r})=2\Delta_0\{\cos Qx+\cos Qy+\cos Qz\} \tag{9}$$

structure, for which the parameter Δ_0 at equilibrium can always be chosen to be real.

Hereafter, where possible, we shall not specify concretely the form of the order parameter. A detailed investigation is possible then only if the phase transition is of second order or close to it. In this case, near the transition, when the order parameter, $\Delta(\mathbf{r})$ is small, we can obtain expressions for all the quantities in the form of an expansion in powers of Δ, similar to the Ginzburg-Landau expansion.

The transition from the normal phase of the ^{3}He–He II solution into the inhomogeneous phase can be observed when the temperature is lowered or, at constant temperature, when the magnetic field is decreased. In the case of a second-order transition, the quantities Δ_m (5) are proportional respectively to $(T_{cH}-T)^{1/2}$ or $(H_c-H)^{1/2}$. The order of the phase transition depends both on the field intensity and on the type of structure (5) of the produced inhomogeneous phase.[1,7,9,11,12]

As a result of the isotropy of the normal phase of the solution, Eqs. (4) specify only the magnitude and not the orientation of the vectors $\mathbf{Q}_m$. The orientation of the structure is determined in this case mainly by the boundary conditions on the helium surface. By analysis of the kernel in the consistency equation,[13] analogous to that carried out for superconductors[3] and superfluid ^{3}He,[14] it can be shown that in the case of specular reflection from the boundary, the most favored orientation will be the one at which the surface is parallel to one of the symmetry planes of the equilibrium order parameter $\Delta(\mathbf{r})$ (5).

3. SUPERFLUID MOTION

We are interested in the spectrum of the low-frequency long-wave oscillations of the system. The corresponding macroscopic equations of motion should include only slowly varying quantities averaged over the small-scale motion at the dimensions of the inhomogeneity of the structure. The macroscopic dynamic variables are usually introduced with the aid of wave-function transformations that do not change the energy. In our case these are the transformations of the phases of the wave functions of the condensates ^{3}He ($\Phi^{(3)}$) and ^{4}He ($\Phi^{(4)}$), which specify two superfluid velocities:

$$v_i^{(3)}=\nabla_i\Phi^{(3)}/2m_3, \quad v_i^{(4)}=\nabla_i\Phi^{(4)}/2m_4, \tag{10}$$

where m_4 is the mass of the ^{4}He atom. The third hydrodynamic variable—the velocity $\mathbf{v}^{(n)}$ of the normal motion—can be introduced with the aid of a Galileo transformation. In the absence of a magnetic field (and in weak fields) these three velocities comprise the entire set of macroscopic variables of three-velocity hydrodynamics of the homogeneous superfluid ^{3}He phase in the ^{3}He–He II solution.[15–20] For inhomogeneous phase it is necessary to introduce additional macroscopic variables connected with the fact that the translations and rotations are not trivial for the order parameter (5). The reaction of the system to inhomogeneous displacements due to elasticity of the structure is investigated in the next section of the paper. On the other hand local violations of the rotational symmetry, which lead to various kinds of flexural oscillations, add as usual to the dispersion law terms with a higher power of the small wave vector than in the phonon modes of the hydrodynamic and elastic oscillations (see below). We therefore do not consider the violation of the rotational symmetry in the present paper. Nor do we touch upon the equations that describe the spin dynamics of the system.

To describe the low-frequency oscillations of the system it suffices to have the hydrodynamic equations in the linear approximation. In our case this circumstance is most important, since the absence of nonlinearity greatly facilitates, for the inhomogeneous phase, the averaging of the rapidly oscillating quantities, and makes it possible to study the reaction of the system to deviations from equilibrium for all the degrees of freedom, independently of one another.

The linearized equations of hydrodynamics can be easily reconstructed with the aid of the usual procedure,[10,11,18] if we know the connection between the mass fluxes and the motion velocities. The hydrodynamic mass fluxes are linear in the velocities, so that we can calculate the proportionality coefficients (the superfluid and normal densities) in succession for the cases when only one of the three velocities is not equal to zero.

We assume first that $\mathbf{v}^{(3)}\neq 0$ and $\mathbf{v}^{(4)}=\mathbf{v}^{(n)}=0$. If the order parameter (5) acquires a time-independent and slowly varying (in space) phase $\Phi^{(3)}$

$$\Phi^{(3)}(\mathbf{r})=\int \Phi_k e^{i\mathbf{k}\mathbf{r}}\frac{d^3k}{(2\pi)^3}$$

then the stationarity condition is specified by the consistency equation (2), which reduces, as a result of the substitutions

$$\Delta\to\Delta\exp(i\Phi^{(3)}), \quad \Delta^+\to\Delta^+\exp(-i\Phi^{(3)}) \tag{11}$$

and after simple transformations, with account taken of (4) and of the hydrodynamic-motion condition $k\ll Q$, to the expressions

$$\frac{\partial^2\Pi}{\partial Q^2}(\mathbf{Q}_m\mathbf{k})^2\Phi_k=0 \tag{12}$$

or, equivalently, to the vanishing of the derivatives of the superfluid velocity $\mathbf{v}^{(3)}$ along the directions of $\mathbf{Q}_m$:

$$\partial(\mathbf{Q}_m\mathbf{v}^{(3)})/\partial(\mathbf{Q}_m\mathbf{r})=0. \tag{12'}$$

Derivatives along the directions perpendicular to $\mathbf{Q}_m$ appear only in the first orders in $\mathbf{k}$. This indicates

that the inhomogeneous rotations of the structure introduce only small corrections to the dispersion of the oscillations.

The ^{3}He mass flux is expressed in the usual manner in terms of the Green's function $G^{\pm}$ of the ^{3}He quasiparticles:

$$j_i(\mathbf{r})=\frac{i}{2}\frac{m_3}{M}T\sum_{s=-\infty}^{\infty}\sum_{(\pm)}\{(\nabla_i^{(2)}-\nabla_i^{(1)})G_{12}^{\pm}\}_{r_1=r_2}+\frac{1}{2}N_3\nabla_i\Phi^{(3)}, \tag{13}$$

where N_3 is the number of the ^{3}He atoms in a unit volume of the solution, and $\Sigma_{(\pm)}$ denotes summation over the values of the projection of the spin of the quasiparticles (of the states G^+ and G^-). The expansion in Δ for the Green's function takes according to (1) the form

$$G_{12}^{\pm}=G_{12}^{\pm(0)}-\int d^3r_3d^3r_4G_{13}^{\pm(0)}\Delta(\mathbf{r}_3)G_{34}^{\mp(0)}\Delta^*(\mathbf{r}_4)G_{42}^{\pm(0)}. \tag{14}$$

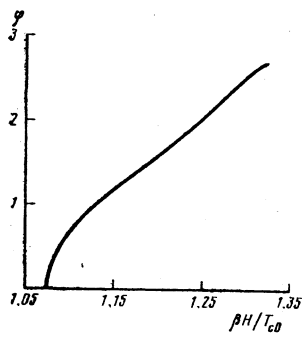
Upon substitution of (11) and (14) in (13), the superfluid flux linearized in the small gradients, for an equilibrium order parameter (5), turns out to be

$$\begin{aligned}\mathbf{j}(\mathbf{r})=&i\frac{m_3}{M}T\sum_{m,n}\Delta_m\Delta_n^*e^{i\mathbf{r}(\mathbf{Q}_m-\mathbf{Q}_n)}\sum_{(\pm)}\sum_{s=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^3}\\ &\times\frac{d^3p}{(2\pi)^3}\Phi_k e^{i\mathbf{k}\mathbf{r}}\mathbf{p}G_p^{\mp}\Big[\mathbf{k}\frac{\partial}{\partial\mathbf{p}}G_{p+Q_m}^{\pm}G_{p+Q_n}^{\pm}+\frac{1}{2}k_ik_j\\ &\times\Big(G_{p+Q_n}\frac{\partial^2}{\partial p_i\partial p_j}G_{p+Q_m}^{\pm}-G_{p+Q_m}^{\pm}\frac{\partial^2}{\partial p_i\partial p_j}G_{p+Q_n}\Big)\Big],\end{aligned} \tag{15}$$

where account is taken of the fact that the presence of the last term in (13) ensures the absence of a superfluid flux in the normal phase at $\Delta=0$. In (15) we have retained the terms proportional to the second derivatives of $\Phi^{(3)}$ since, owing to the rapidly oscillating factors $\exp\{i\mathbf{r}\cdot(\mathbf{Q}_m-\mathbf{Q}_n)\}$, such quantities can in principle make contributions to div j of the same order as the terms linear in k.

It is simpler to calculate div j directly, all the more since it is precisely such quantities which enter in the hydrodynamic equations. An expression for div j is obtained from (15) by differentiating with respect to r and integrating with respect to the momenta. The term linear in k then vanishes. As a result of long transformations, the expressions for div j takes the form

$$\begin{aligned}\operatorname{div}\mathbf{j}=&-\frac{1}{4}\frac{m_3}{M}N_3\{\varphi(H)\}^2\sum_{m,n}\frac{\Delta_m\Delta_n^*}{(\beta H)^2}\exp\{i\mathbf{r}(\mathbf{Q}_m-\mathbf{Q}_n)\}\\ &\times\int\frac{d^3k}{(2\pi)^3}\Phi_k\frac{(\mathbf{k}\mathbf{Q}_m)^2+(\mathbf{k}\mathbf{Q}_n)^2}{Q^2},\end{aligned} \tag{16}$$



where

$$\Big\{\varphi\Big(\frac{\beta H}{T_{c0}}\Big)\Big\}^2=6\sum_{\nu=1}^{\infty}\frac{h_\nu^2}{2\nu-1}\frac{1}{q}\Big\{\frac{q+h}{[1+(h_\nu+q_\nu)^2]^2}+\frac{q-h}{[1+(h_\nu-q_\nu)^2]^2}\Big\},$$
$$h=\frac{\beta H}{T_{cH}},\quad q=\frac{Qv_F}{2T_{cH}},\quad h_\nu=\frac{h}{\pi(2\nu-1)},\quad q_\nu=\frac{q}{\pi(2\nu-1)}. \tag{17}$$

A plot of $\varphi(H)$, obtained by numerical calculation using the data of Ref. 1 on $T_{cH}(H)$ and $Q(H)$, is shown in the figure. Assuming that the motion specified by the consistency equations (12) is stationary, the expression for div j, as expected, vanishes.

The value of div j (16) is already proportional to the second derivative of a slowly varying quantity—the phase $\Phi^{(3)}$ of the wave function of the condensate. Therefore averaging this expression over small dimensions of the order of $1/Q$ entails no difficulty. The final form of the averaged expression for div j at $\mathbf{v}^{(3)}\neq0$ and $\mathbf{v}^{(4)}=\mathbf{v}^{(n)}=0$ is

$$\operatorname{div}\mathbf{j}^{(3)}=\rho^{(s)}\sum_m\frac{|\Delta_m|^2}{\Delta_A^2}\frac{\partial(\mathbf{Q}_m\mathbf{v}^{(3)})}{\partial(\mathbf{Q}_m\mathbf{r})}, \tag{18}$$

where the superfluid density $\rho^{(s)}$ is

$$\rho^{(s)}=\frac{m_3^2}{M}N_3\Big(\frac{\Delta_A}{\beta H}\Big)^2\varphi^2(H),\qquad \Delta_A^2=\sum_m|\Delta_m|^2. \tag{19}$$

The superscript of the flux $\mathbf{j}^{(3)}$ in (18) shows that in this case we are dealing with the ^{3}He mass flux. The total mass flux in the solution differs from $\mathbf{j}^{(3)}$ by a factor M/m_3. Thus, the ^{4}He mass flux at $\mathbf{v}^{(3)}\neq0$ and $\mathbf{v}^{(4)}=\mathbf{v}^{(n)}=0$ is

$$\operatorname{div}\mathbf{j}^{(4)}=\operatorname{div}(\mathbf{j}-\mathbf{j}^{(3)})=\Big(\frac{M}{m_3}-1\Big)\rho^{(s)}\sum_m\frac{\partial(\mathbf{Q}_m\mathbf{v}^{(3)})}{\partial(\mathbf{Q}_m\mathbf{r})}\frac{|\Delta_m|^2}{\Delta_A^2}. \tag{20}$$

The mass fluxes in the remaining cases can be determined, with practically no calculations whatever, by following the method developed by Andreev and Bashkin[18] for the homogeneous superfluid phase of the solution. The onset of superfluid motion with velocity $\mathbf{v}^4$ ($\mathbf{v}^{(3)}=\mathbf{v}^{(n)}=0$) leads to the appearance in the Hamiltonian of the ^{3}He quasiparticles of an additional term[10,11,21]

$$^1/_2(1-m_3/M)(\hat{\mathbf{p}}\mathbf{v}^{(4)}+\mathbf{v}^{(4)}\hat{\mathbf{p}}) \tag{21}$$

($\hat{\mathbf{p}}$ is the momentum of the ^{3}He quasiparticles), meaning that additional terms proportional to gradients of $\Phi^{(4)}$ appear in (1). On the other hand, a change of the ^{3}He quasiparticle wave functions, $\Psi\to\Psi\exp(i\Phi^{(3)}/2)$ would lead to the appearance of analogous quantities apart from the substitution $(1-m_3/M)\Phi^{(4)}-(m_3/M)\Phi^{(3)}$. As a result, the response of the system of ^{3}He quasiparticles to the onset of $\mathbf{v}^{(4)}$ is described by an expression similar to (18):

$$\operatorname{div}\mathbf{j}^{(3)}=\Big(\frac{M}{m}-1\Big)\rho^{(s)}\sum_m\frac{|\Delta_m|^2}{\Delta^2}\frac{\partial(\mathbf{Q}_m\mathbf{v}^{(4)})}{\partial(\mathbf{Q}_m\mathbf{r})}. \tag{22}$$

The ^{4}He mass flux is easily determined in this case from a comparison of the total momentum (mass flux)

$$\mathbf{j}=\rho_4\mathbf{v}^{(4)}+\sum_{\mathbf{p}}\mathbf{p}n_{\mathbf{p}}$$

and from the definition of the ^{3}He quasiparticle current

$$\mathbf{j}^{(3)}=m_3\sum_{\mathbf{p}}\frac{\partial\hat{\varepsilon}}{\partial\mathbf{p}}n_{\mathbf{p}}$$

($\hat{\varepsilon}$ is the energy of the ^{3}He quasiparticles, and $n_{\mathbf{p}}$ is the particle-number operator). When account is taken of (21) and (22), these expressions yield the following value of the ^{4}He mass flux:

$$\operatorname{div}\mathbf{j}^{(4)}=\operatorname{div}(\mathbf{j}-\mathbf{j}^{(3)})=(\rho_4-MN_3+m_3N_3)\operatorname{div}\mathbf{v}^{(4)}+\left(\frac{M}{m_3}-1\right)^2\rho^{(s)}\sum_m\frac{|\Delta_m|^2}{\Delta_A{}^2}\frac{\partial(\mathbf{Q}_m\mathbf{v}^{(4)})}{\partial(\mathbf{Q}_m\mathbf{r})},\qquad(23)$$

where ρ_4 is the density of ^{4}He.

The reaction of the system to the normal motion can be easily ascertained with the aid of the Galilean-invariance relations. Inasmuch as at $\mathbf{v}^{(n)}=\mathbf{v}^{(3)}=\mathbf{v}^{(4)}$ the mass fluxes of ^{3}He and ^{4}He are respectively equal to $m_3N_3\mathbf{v}^{(n)}$ and $\rho_4\mathbf{v}^{(n)}$, we obtain, taking (18)–(23) into account, the following expressions:

$$\operatorname{div}\mathbf{j}^{(3)}=m_3N_3\operatorname{div}\mathbf{v}^{(n)}-\frac{M}{m_3}\rho^{(s)}\sum_m\frac{|\Delta_m|^2}{\Delta_A{}^2}\frac{\partial(\mathbf{Q}_m\mathbf{v}^{(n)})}{\partial(\mathbf{Q}_m\mathbf{r})},\qquad(24)$$

$$\operatorname{div}\mathbf{j}^{(4)}=(M-m_3)N_3\operatorname{div}\mathbf{v}^{(n)}-\left(\frac{M^2}{m_3{}^2}-\frac{M}{m_3}\right)\rho^{(s)}\sum_m\frac{|\Delta_m|^2}{\Delta_A{}^2}\frac{\partial(\mathbf{Q}_m\mathbf{v}^{(n)})}{\partial(\mathbf{Q}_m\mathbf{r})}.\qquad(25)$$

Formulas (18)–(25) specify completely the connection between the velocities of the macroscopic motion and the mass fluxes, and by the same token determine the equations of the hydrodynamics of the inhomogeneous superfluid phase of the solution (see Sec. 5).

4. ELASTICITY OF STRUCTURE

For the inhomogeneous phase, we can introduce one other dynamic variable $\mathbf{u}(\mathbf{r},t)$, connected with the invariance of the order parameter (5), in contrast to the Hamiltonian, to displacements $\mathbf{r}\to\mathbf{r}+\mathbf{u}$. Correspondingly, an additional low-frequency mode can appear, characterizing the elastic oscillations in the system. Rigid-body elastic modes of this kind exist also in other systems with inhomogeneous condensation,[12,22] but the spectrum of the oscillations in a solution can be exactly and consistently studied. The displacement u, obviously, is meaningful only for those directions for which the transformation $\Delta(\mathbf{r})\to\Delta(\mathbf{r}+\mathbf{u})$ of the order parameter is not an identity transformation. Thus, for the layered phase (8) we can introduce only one component, parallel to $\mathbf{Q}$, of the displacement vector u.

The displacement u is not a completely independent new variable. The quantity $\dot{\mathbf{u}}$ defines the transformation $\mathbf{r}\to\mathbf{r}+\mathbf{u}t$ and is the general (Galilean) velocity of the motion. The flow velocity can be expressed in trivial fashion, with the aid of the flux-invariance relations, in terms of the velocity of the normal motion. In our case, however, we are interested in the region near the superfluid transition, when the difference between the total and normal densities of ^{3}He is small, and at the same accuracy with which the relations obtained above for the superfluid and normal densities are valid, we have for the hydrodynamic equations

$$\mathbf{u}=\mathbf{v}_n\qquad(26)$$

for all directions for which u is defined.

We are interested in the reaction of the system to a time-constant weakly inhomogeneous deformation ($k\ll Q$)

$$\mathbf{u}(\mathbf{r})=\int\mathbf{u}_{\mathbf{k}}e^{i\mathbf{k}\mathbf{r}}\frac{d^3k}{(2\pi)^3}.$$

The wave-function transformation $\Psi(\mathbf{r})\to\Psi[\mathbf{r}+\mathbf{u}(\mathbf{r})]$ adds to the right-hand side of the equations (1), linearized in the gradients, the small quantities

$$-M^{-1}(\nabla_i^{(1)}u_k(\mathbf{r}_1))\nabla_i^{(1)}\nabla_k^{(1)}G_{12}^{\pm};\quad -M^{-1}(\nabla_i^{(1)}u_k(\mathbf{r}_1))\nabla_i^{(1)}\nabla_k^{(1)}F_{12}.$$

Since the displacement enters in these expressions only in a combination of the form $[\nabla_i^{(1)}u_k(\mathbf{r}_1)]\nabla_i^{(1)}\nabla_k^{(1)}$, all the answers, as expected, will contain the symmetrical strain tensor $u_{ik}=(\partial u_i/\partial r_k+\partial u_k/\partial r_i)/2$. For the linearized dynamics equations of the system, the responses to small deviations from equilibrium are calculated for all the degrees of freedom independently of one another. This means that the reaction of the system to an inhomogeneous deformation can be calculated at a fixed phase of the wave function of the condensate.

The stationarity condition is specified by the consistency equation

$$\sum_{s=-\infty}^{\infty}\int G_{12}^{\mp(0)}\{u_{ik}(\mathbf{r}_2)\nabla_i^{(2)}\nabla_k^{(2)}G_{23}^{\mp(0)}\Delta^*(\mathbf{r}_3)G_{31}^{\pm(0)}-\Delta^*(\mathbf{r}_2)G_{23}^{\pm(0)}u_{ik}(\mathbf{r}_3)\nabla_i^{(3)}\nabla_k^{(3)}G_{31}^{\pm(0)}\}d^3r_2d^3r_3=0,$$

which reduces at $k\ll Q$, after prolonged transformations, to the form [cf. (12)]

$$Q\partial^2(\mathbf{Q}_m\mathbf{u})/\partial(\mathbf{Q}_m\mathbf{r})^2=0.\qquad(27)$$

Here, just as in (12), the contribution of the inhomogeneous rotations, i.e., from the deformations along directions perpendicular to $\mathbf{Q}_m$, arises only in the first orders in k.

The substitution $\mathbf{r}\to-\mathbf{r}$ reverses the directions of the mass fluxes, while the strain tensor u_{ik} does not reverse sign. Therefore, if the group of transformations that leave the order parameter (5) invariant contains an inversion, then, in the linear approximation, the inhomogeneous deformation does not cause a mass flux proportional to u_{ik}. This statement, which can be easily verified by direct calculation, is valid for all the symmetrical phases (7) that are of principal physical interest. For the asymmetrical phase (6), the mass flux proportional to u_{ik} differs from zero. The corresponding expression for the current can be easily obtained by noting that when the superfluid velocity $\mathbf{v}^{(3)}$ and the strain u_{ik} is introduced, the only change in the transformations of the order parameter for this phase are different is the replacement $\Phi^{(3)}\to\mathbf{Q}\cdot\mathbf{u}$. Therefore the expression for the mass flux under inhomogeneous deformation of the structure (6) coincides with (18) when $\Phi^{(3)}$ is replaced by $\mathbf{Q}\cdot\mathbf{u}$ and there is no summation over the harmonics of the order parameter.

For symmetrical phases, only the appearance of the additional momentum flux Π_{ik} is connected with the inhomogeneous deformation. The expression for Π_{ik} can be obtained with the aid of cumbersome calculations with the Green's function (14), in the same manner the

mass flux was determined in the preceding section. However, the same expression for the average flux Π_{ik} can be obtained practically without calculations by starting from the following simple considerations.

The energy of the system near the transition is a bilinear form of the order parameter and contains quantities of the type $\Delta\Delta^*$. Its derivatives with respect to the superfluid velocity and with respect to the strain tensor specify respectively the mass flux and the momentum flux. On the other hand, variation of the quantity $\Delta\Delta^*$ with respect to the superfluid velocity and to the strain tensor differs only in that the phase Φ is replaced by quantities of the form $\mathbf{Q}\cdot\mathbf{u}$. Therefore the energy increment due to the inhomogeneous deformation

$$\sum_{m,n}(\mathbf{j}^{(m)}\mathbf{k})(\mathbf{Q}_n\mathbf{u})+(\mathbf{j}^{(n)}\mathbf{k})(\mathbf{Q}_n\mathbf{u})$$

is expressed in terms of the currents $\mathbf{j}^m$ that are the response of the system to the change of the phases Φ_m of the harmonics of the order parameter (5). The values of $\mathbf{j}^{(m)}$ are given here by the integrals (15) with $\Phi_m=\mathbf{Q}_m\cdot\mathbf{u}$ (in the approximation linear in $\mathbf{k}$). A simple investigation of these integrals shows that in the expression for the elastic moduli, in each of the order-parameter-harmonic term proportional to $\exp\{i(\mathbf{Q}_m-\mathbf{Q}_n)\cdot\mathbf{r}\}$, only the diagonal components along the vectors $\mathbf{Q}_m+\mathbf{Q}_n$ differ from zero. The calculation of the remaining integrals, naturally, leads to an expression similar to (18) for the averaged value of the quantity

$$\frac{\partial\Pi_{ik}}{\partial r_k}=\frac{\rho^{(s)}}{m_3^2}Q^2\sum_m\frac{|\Delta_m|^2}{\Delta_A^2}(Q_m)_i\frac{\partial^2(\mathbf{Q}_m\mathbf{u})}{\partial(\mathbf{Q}_m\mathbf{r})^2}. \tag{28}$$

The quantity Π_{ik} (28), which is proportional to Q^2, is quite small. This means that the propagation velocity of the elastic oscillations is exponentially small in terms of the concentration of the solution. However, only the spin-averaged elastic moduli are so small. The elastic moduli that are anti-symmetrized with respect to the spins contain, in comparison with (28), the large factor p_F/Q.

5. LOW-FREQUENCY OSCILLATIONS

The spectrum of the long-wave acoustic oscillations is determined by the linearized hydrodynamics equations. The hydrodynamics equations for the inhomogeneous superfluid phase are derived in the usual manner from the conservation laws, with account taken of the relations obtained above between the fluxes and the dynamic variables. An analysis of the conservation laws, similar to that carried out in Refs. 15 and 18 for the homogeneous superfluid phase of the solution, leads to the following system of linearized equations for the averaged hydrodynamic quantities:

$$\frac{\partial\rho_3}{\partial t}+\operatorname{div}\mathbf{j}^{(3)}=0,\quad \frac{\partial\rho_4}{\partial t}+\operatorname{div}\mathbf{j}^{(4)}=0,$$
$$\frac{\partial(\mathbf{j}_i^{(3)}+\mathbf{j}_i^{(4)})}{\partial t}+\frac{\partial\Pi_{ik}}{\partial r_k}=0,\quad \frac{\partial S}{\partial t}+S\operatorname{div}\mathbf{v}=0, \tag{29}$$
$$\frac{\partial\mathbf{v}^{(3)}}{\partial t}+\nabla\mu_3=0,\quad \frac{\partial\mathbf{v}^{(4)}}{\partial t}+\nabla\mu_4=0,\quad \frac{\partial\mathbf{u}}{\partial t}=\mathbf{v}=\mathbf{v}_n,$$

where

$$\mathbf{j}^{(3)}=m_3N_3\mathbf{v}+\rho^{(s)}\sum_m\frac{|\Delta_m|^2}{\Delta_A^2}\mathbf{e}_m\left[\mathbf{e}_m\mathbf{v}^{(3)}+\left(\frac{M}{m_3}-1\right)\mathbf{e}_m\mathbf{v}^{(4)}\right],$$
$$\mathbf{j}^{(4)}=(M/m_3-1)\mathbf{j}^{(3)}+(\rho_4-MN_3+m_3N_3)\mathbf{v}^{(4)}, \tag{30}$$
$$\frac{\partial\Pi_{ik}}{\partial r_k}=\frac{\partial P}{\partial r_i}+\frac{\rho^{(s)}}{m_3^2}Q^2\sum_m\frac{|\Delta_m|^2}{\Delta_A^2}(e_m)_i\partial^2(\mathbf{ue}_m)/\partial(\mathbf{re}_m)^2.$$

The unit vectors $\mathbf{e}_m=\mathbf{Q}_m/Q$ characterize here the anisotropy of the structure (7), while ρ_3,ρ_4 and μ_3,μ_4 are respectively the densities and chemical potentials of ^{3}He and ^{4}He in the solution, S is the entropy per unit volume, and P is the pressure. The corresponding thermodynamic identity is of the form

$$dP=\rho_3d\mu_3+\rho_4d\mu_4+SdT.$$

The system (29)–(30) already takes into account the fact that all the calculations are carried out near the superfluid transitions, when the difference between the total and normal densities of the impurity component and between the general (Galilean) velocity and the normal velocity is negligibly small. All the results obtained with the aid of these equations are meaningful only in the principal order in $\rho^{(s)}$.

The system (29)–(30) differs from the equations of the three-velocity hydrodynamics of the homogeneous superfluid phase ^{3}He in a ^{3}He–He II solution in the substantial anisotropy and in the presence, in the expression for the momentum flux tensor, of an additional term characterizing the elasticity of the structure. In the homogeneous phase of the solution, in the absence of a magnetic field, three types of acoustic oscillations can propagate[18]: of the density, of the concentration, and of the temperature. These waves can propagate also in the inhomogeneous phase of the solution. In addition, the hydrodynamics equations for the inhomogeneous phase have also distinct solutions that describe the elastic oscillations (of the rigid-body type) of the structure.

The elasticity of the structure has practically no effect on the density and concentration oscillations, since the corresponding correction would be of the order of Q^2 and would therefore be exponentially small in terms of the concentration. Nor are the oscillations of this type influenced by the anisotropy of Eqs. (29) and (30), since the oscillations have at any rate a low sensitivity to the superfluid properties of the impurity component.[11,18] In the inhomogeneous phase, just as in the absence of a field, the propagation velocity of the density oscillations is close to the speed of sound in pure ^{4}He, and the oscillations of the concentration propagate with an approximate velocity $v_F/\sqrt{3}$.

The appearance of inhomogeneity of the structure influences noticeably the slower oscillations, whose propagation velocity is exponentially small in terms of the concentration of the ^{3}He in the solution. In this case substantial changes occur in the propagation velocities of the temperature waves, and additional elastic-oscillation modes appear. Since the propagation velocity of these oscillations $c\ll v_F$, to determine the dispersion law of the temperature and elastic oscillations it is necessary to put in Eqs. (29) $\partial\rho_3/\partial t=\partial\rho_4/\partial t=0$. As a result, Eqs. (29) and (30) reduce to the system of equations $a_{ik}v_k=0$ with a matrix a_{ik} equal to

$$a_{ik}=-\delta_{ik}+n_k\sum_m\frac{|\Delta_m|^2}{\Delta_A^2}(e_m)_i(\mathbf{n}e_m)\left\{\sum_m\frac{|\Delta_m|^2}{\Delta_A^2}(\mathbf{n}e_m)^2\right\}^{-1}$$
$$+\frac{\rho^{(s)}Q^2}{m_3^2MN_3c^2}\sum_m\frac{|\Delta_m|^2}{\Delta_A^2}(e_m)_i(e_m)_k+\frac{n_in_k}{MN_3c^2}\Big\{1 \qquad (31)$$
$$-\frac{c^2}{c_T^2}\left[\sum_m\frac{|\Delta_m|^2}{\Delta_A^2}(\mathbf{n}e_m)^2\right]^{-1}\Big\}\Big/\left\{\frac{1}{S}\frac{\partial S}{\partial P}+\frac{\rho^{(s)}}{m_3^2N_3^2c_T^2}\right\}.$$

Here n is a unit vector along the direction of the wave vector **k** of the oscillations, $\mathbf{n}=\mathbf{k}/k, c=\omega/k$ is the propagation velocity of the oscillations, ω is the frequency, $\mathbf{e}_m=\mathbf{Q}_m/Q$, and the quantity c_T

$$c_T^2=(TS^2/C)(\rho^{(s)}/m_3^2N_3^2),$$

coincides formally with the velocity of the temperature waves in the homogeneous phase of the solution ($C=T\partial S/\partial T$ is the heat capacity per unit volume of the solution).

The oscillation propagation velocity c is determined from the dispersion equation

$$\mathrm{Det}(a_{ik})=0. \qquad (32)$$

This equation for the velocity has solutions proportional to the square root of the small quantity $\rho^{(s)}$. In this case the principal term in the matrix a_{ik} (31) is the last one, which is inversely proportional to $\rho^{(s)}$. This makes it possible to obtain directly an expression for the propagation velocity of the temperature waves in the inhomogeneous superfluid phase of this solution (cf. Refs. 11 and 18):

$$c^2=c_T^2\sum_m\frac{|\Delta_m|^2}{\Delta_A^2}\alpha_m^2,\quad \alpha_m=\mathbf{n}e_m. \qquad (33)$$

The dispersion equation (32) for the inhomogeneous phase, however, has also other solutions of the same order, which determine the propagation velocity of the elastic oscillations. The expression for the corresponding velocities, in the principal order in $\rho^{(s)}$ has the rather cumbersome form

$$c^2=\frac{\rho^{(s)}Q^2}{2m_3^2MN_3}\left\{\sum_m\frac{|\Delta_m|^2}{\Delta_A^2}\alpha_m^2(1-\alpha_m^2)\right.$$
$$\left.\pm\left[\sum_{m,n}\frac{|\Delta_m\Delta_n|^2}{\Delta_A^4}\alpha_m^2\alpha_n^2\{2(\nu_{mn}-\alpha_m\alpha_n)^2-(1-\alpha_m^2)(1-\alpha_n^2)\}\right]^{1/2}\right\}; \qquad (34)$$
$$\nu_{mn}=\mathbf{e}_m\mathbf{e}_n,\quad \alpha_m=\mathbf{n}e_m.$$

The propagation velocity of the elastic waves, just as that of the temperature waves, is exponentially small in the concentration of the solution and depends substantially on the mutual orientation of the wave vector of the oscillations and of the vectors $\mathbf{e}_m$, which characterize the structure. For example, in the case of the layered phase (8), the solutions (33) and (34) of the dispersion equation (32) for the temperature and elastic oscillations take the respective form

$$c^2=c_T^2\alpha^2,\quad c^2=\frac{\rho^{(s)}Q^2}{m_3^2MN_3}\alpha^2(1-\alpha^2);\quad \alpha=\mathbf{k}\mathbf{Q}/kQ.$$

Thus, in contrast to the homogeneous superfluid phase, the spectrum of the slow long-wave oscillations of the inhomogeneous phase is anisotropic and includes additional modes of other oscillations of the structure. The dependence of the propagation velocity of the elastic (and temperature) oscillations on the intensity of the external magnetic field is determined by the behavior $\rho^{(s)}(H)$ (19) and of $Q(H)$.[1,11]

The long-wave hydrodynamic oscillations investigated in the present paper do not include the entire spectrum of the possible low-frequency waves propagating in the spatially-inhomogeneous phase ^{3}He in the ^{3}He–He II solution. In principle, oscillations can exist corresponding to inhomogeneous rotations of the structure, and also low-frequency oscillations with large (of the order of Q_m-Q_n) wave vectors. Great interest attaches also to the study of the additional modes that characterize the spin dynamics of the system.

Many of the results obtained in the present paper are not restricted to ^{3}He–^{4}He solutions and can be easily applied to cases of other types of Fermi systems with inhomogeneous pairing. In particular, the dispersion laws for the temperature and elastic oscillations take a form analogous to (33) and (34) also for other inhomogeneous Fermi systems near a second-order transition, independently of the presence of a superfluid Bose background.

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