NMR and superfluidity of ^{3}He in ^{3}He–^{4}He solutions

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Two possibilities for an NMR determination of the temperature of the superfluid transition of ^{3}He in a ^{3}He–^{4}He solution are discussed: measuring the spin diffusion coefficient in weak magnetic fields at ultralow temperatures, and measuring the ratio of the spin diffusion coefficient to the spin-wave absorption coefficient in strong magnetic fields, at temperatures which are not very low. The transition temperature is estimated from the experimental data available. A study has been made of the effect of the superfluid transition in the ^{3}He quasiparticle system on the propagation of transverse spin waves and longitudinal spin-sound waves in ^{3}He–^{4}He solutions. There is a range of weak magnetic fields, bounded at both ends, in which weakly damped spin-sound waves can propagate.

1. INTRODUCTION

One of the most interesting problems in ultralow-temperature physics is observing the superfluidity of ^{3}He in a ^{3}He–^{4}He solution. All previous attempts to observe superfluidity in a system of ^{3}He impurity quasiparticles in solutions have been unsuccessful,[1–4] despite cooling of the solutions below 1 mK. As a result, the ^{3}He–^{4}He solution remains the only liquid with a significant entropy and heat capacity at $T<1$. If the temperature of the superfluid transition in the system of ^{3}He quasiparticles turns out to be significantly below 100 μK, then any attempt to observe this transition at the present state of the art in low-temperature techniques will be hopeless.

Experience in work at ultralow temperatures, in particular, in research on superfluid ^{3}He, has shown that the most promising methods for studying ^{3}He–^{4}He solutions at ultralow temperatures are NMR methods. In the following section of this paper we discuss how a reliable estimate of the temperature (T_c) of the superfluid transition in the system of ^{3}He impurity quasiparticles can be extracted from NMR data on a solution. Apparently the most convenient way to observe this transition would be to measure the temperature dependence of the susceptibility or that of spin-dynamics parameters. In the last two sections of this paper we accordingly examine the propagation of transverse spin waves (Section 3) and of longitudinal spin-sound waves (Section 4) when superfluidity is present in a system of ^{3}He impurity quasiparticles.

In the absence of a ^{3}He superfluidity, spin waves of two types can propagate in a ^{3}He–HeII solution in a magnetic field: transverse (Silin) spin waves with a quadratic dispersion law $\omega \propto k^2$ and high-frequency longitudinal Fermi-liquid spin waves with a linear dispersion law.[5] Clearly, the superfluidity of ^{3}He in solution, at least near the transition, will have essentially no effect on the transverse spin waves; it could only give rise to some change in the coefficient of k^2 in the spectrum. The situation regarding longitudinal spin waves is slightly more complicated. The propagation velocity of longitudinal spin waves in solution, like that of any high-frequency Fermi-liquid oscillations in slightly nonideal Fermi systems, is exponentially close (as a function of ^{3}He concentration) to the Fermi velocity, so that there is a comparatively strong collisionless damping of the waves, even at low temperatures. For systems of this sort, with a weak Fermi-liquid interaction, the appearance of superfluidity is known (Refs. 6 and 7, for example) to cause a sharp increase in the collisionless damping and to make it essentially impossible to observe such waves at any temperature below T_c. The situation changes slightly, however, when a magnetic field is imposed. On one hand, a magnetic field causes the wave propagation velocity to approach the Fermi velocity (at a fixed temperature, this would mean an increase in the damping), but on the other hand the field sharply reduces the transition temperature $T_c(H)$. A question which arises here is whether there is a temperature region $T \gtrsim T_c$ near the transition in which weakly damped longitudinal spin waves can propagate. Furthermore, ^{3}He–^{4}He solutions differ from ordinary slightly nonideal Fermi systems in that longitudinal spin waves are accompanied by oscillations of the density of the HeII Bose background in an external magnetic field. This effect will also be manifested significantly in the conditions for the propagation and damping of the spin waves.

2. SPIN DYNAMICS IN ^{3}He–^{4}He SOLUTIONS; TEMPERATURE OF THE SUPERFLUID TRANSITION

At present there is a very large scatter in the estimates of the temperature at which ^{3}He goes into a superfluid state in a ^{3}He–^{4}He solution. At comparatively high ^{3}He concentrations, the estimates for T_c range from 10^{-3} to 10^{-8} K and are thus somewhat reminiscent of the scatter in the estimates of T_c for pure ^{3}He before the transition was observed. This scatter in the estimates should not be surprising since the transition temperature T_c is exponentially small as a function of the ^{3}He concentration in comparison with the degeneracy temperature (T_0) of ^{3}He in solution. (The limiting concentration of ^{3}He in solution in the limit $T \to 0$ at the saturation vapor pressure is about 6.5%.) The argument of the corresponding exponential function is determined by the effective attraction of ^{3}He quasiparticles. However, we do not have a systematic elementary theory which describes the

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interaction of ^{3}He in a solution with an arbitrary ^{3}He concentration. Studies of solutions thus usually resort to one of a variety of model or phenomenological descriptions of the interaction, with parameter values determined from a comparison with experimental data on the thermodynamics and kinetics of the solutions. The use of various descriptions of the interaction—descriptions which have not yet been completely justified at the elementary level—has the consequence that the parameter values for the interaction which enter the argument (large in magnitude) of the exponential function for T_c differ significantly from model to model, with the further consequence that there is a huge scatter in the estimates of T_c.

Regarding the transition of ^{3}He in solution to a superfluid state it is possible to make one precise assertion: At sufficiently low temperatures and ^{3}He concentrations, the system of ^{3}He quasiparticles in solution is a dilute degenerate gas of slow fermions, for which the BCS theory gives not a model description but an exact description of the superfluid transition.[5] Some questions which remain open are the exact value of the s-scattering length of the ^{3}He quasiparticles, which determines the coupling constant (the argument of the exponential function for T_c) in the BCS theory; the maximum ^{3}He concentration at which the BCS theory remains valid for calculating T_c; and the transition temperature at higher concentrations. The first of these questions is discussed in the present section of the paper.

The s-scattering length of ^{3}He quasiparticles in solution, a, should be determined from experimental data on the characteristics of dilute solutions at temperatures low enough that the number of phonons and rotons in the system is negligible, and the ^{3}He impurity quasiparticles are the only excitations of the system; the HeII is a superfluid Bose background (a physical vacuum). It is not very convenient to extract the values of a from thermodynamic measurements. The reason is that for the thermodynamic parameters of dilute solutions the interaction of ^{3}He quasiparticles yields only small corrections (for the nonideal nature of the situation) to the thermodynamic characteristics of a nearly ideal gas of impurity quasiparticles. The value of a can be found most accurately from the values of the transport coefficients in the gas of ^{3}He impurity quasiparticles; these coefficients are proportional to $1/a^2$ at a sufficiently low ^{3}He concentration. In principle, suitable experimental data for this purpose would be the values of the viscosity coefficient η, the thermal conductivity $\varkappa$, and the spin diffusion coefficient for both degenerate $T \ll T_0$ and nondegenerate (Boltzmann; $T \gg T_0$) solutions (in the intermediate temperature region, there have been essentially no calculations of the kinetic coefficients). For an accurate determination of a we need to use the results of measurements for solutions with a very low ^{3}He concentration, $x \lesssim 10^{-3}$, for which the values of the kinetic coefficients can be calculated reliably in the kinetic theory of gases (Ref. 8, for example).

For such dilute solutions, however, attempts to measure and interpret data in the degenerate and Boltzmann regions run into some specific difficulties. In the Boltzmann region the primary difficulty in interpreting experimental data on the kinetics of dilute solutions is that collisions of ^{3}He quasiparticles with phonons and rotons are still playing a significant role at comparatively high temperatures $T \gg T_0$: If the ^{3}He quasiparticles collided with each other at $T \gg T_0$, the kinetic coefficients of the solution vary as the square root of the temperature, η, $\varkappa$, $D \propto T^{1/2}$ (Ref. 8, for example). Experimentally, in contrast, a square-root dependence $T^{1/2}$ is essentially not observed over any significant temperature interval (see, for example, the data[9] on the viscosity η).

For degenerate solutions, $T \ll T_0$, the difficulties stem primarily from the low value of the degeneracy temperature $T_0 = p_0^2/2M$ [$p_0 = (3\pi^2 N_3)^{1/3}\hbar$ and M are the Fermi momentum and effective mass of the ^{3}He quasiparticles in the solution, and N_3 is the number of ^{3}He particles per unit volume of the solution; at saturation vapor pressure we would have $M \approx 2{,}3 m_3$, where m_3 is the mass of the ^{3}He atom] at a low ^{3}He concentration in the solution, x (numerically, $T_0 \approx 2{,}6 x^{2/3}$ K). This means that for $x \lesssim 10^{-3}$ measurements would have to be carried out at ultralow temperatures, and over a fairly broad temperature interval, since in this temperature region the only possibility for verifying the thermometry (the equality of the phonon and impurity temperatures) and the presence of degeneracy is to observe the characteristic temperature dependence $(T_0/T)^2$ for measured transport coefficients. To the best of our knowledge, there has been essentially no systematic study of transport phenomena at such low temperatures and ^{3}He concentrations.

There is an alternative way to determine a, without carrying out measurements at extremely low temperatures. This method is based on the occurrence of the Leggett-Rice effect[10] in a Fermi system in a magnetic field at low temperatures. In a magnetic field, the spin dynamics of Fermi system is known to be described by two parameters: the spin diffusion coefficient D and the dimensionless parameter $\Omega_{\text{int}}\tau$ (Ω_{int} is the typical precession frequency of the magnetization in the molecular field, τ is the exchange relaxation time, and the quantity $1/\Omega_{\text{int}}\tau$ determines the damping of spin waves; the parameter $\Omega_{\text{int}}\tau$ is frequently denoted as μM, as in Ref. 10). The equation of the spin dynamics in the case of a low-density Fermi gas is essentially the same in form for any degree of degeneracy of the gas.[11] The parameters D and $\Omega_{\text{int}}\tau$ are easily found from pulsed NMR experiments, through measurements of (for example) the dependence of the amplitude of a spin-echo signal on the time delay between pulses. If the parameter $\Omega_{\text{int}}\tau$ is to be comparatively large, so that it can be measured accurately, it is necessary to lower the temperature or raise the magnetic field $\mathbf{H}$. Information on D or $\Omega_{\text{int}}\tau$ independently is not very informative for solutions with $x \lesssim 10^{-3}$ since, as we mentioned earlier, the phonon contribution to D and τ may be significant at $T \gtrsim T_0$, while the region $T \ll T_0$ is difficult to reach. Furthermore, in strong magnetic fields the diffusion rates and the relaxation times in the directions along and across the magnetic field $\mathbf{H}$ are very different. While the properties $D_\perp$ and $\Omega_{\text{int}}\tau_\perp$ are measured in pulsed NMR measurements, all of the calculations which have been carried out refer exclusively to $D_\parallel$ and $\Omega_{\text{int}}\tau_\parallel$ (Ref. 8). All of these difficulties fade away, however, if the ratio $\Omega_{\rm int}\tau_\perp/D_\perp$ is extracted from the experiment. According to Refs. 8 and 11, for a low-density, low-temperature gas of fermions under the conditions

$$T,\ T_0 \ll \hbar^2/Ma^2, \qquad (1)$$

the ratio $\Omega_{\rm int}\tau_\perp/D_\perp$ is

$$\frac{\Omega_{\rm int}\tau_\perp}{D_\perp} = -\frac{12\pi a\hbar}{M}\frac{(N_+-N_-)^2}{N_+\langle v^2\rangle_+ - N_-\langle v^2\rangle_-} = -\frac{4\pi a\hbar(N_+-N_-)}{M}\begin{cases} M/T, & T\gg T_0,\\ \dfrac{5(N_+-N_-)}{N_+v_+{}^2-N_-v_-{}^2}, & T\ll T_0,\end{cases} \qquad (2)$$

where $N_\pm$ and $v_\pm$ are the numbers of fermions per unit volume and the Fermi velocities for ^{3}He quasiparticles with spin projections $\pm 1/2$ on to the field direction, $\mathbf{e} = \mathbf{H}/H$, and $\langle v^2\rangle_\pm$ is the average square velocity of the fermions, found from $n_\pm$, the distribution functions of particles with a certain spin projection in an ideal gas:

$$\langle v^2\rangle_\pm = \frac{1}{N_\pm}\int v^2 n_\pm \frac{d^3\mathbf{p}}{(2\pi\hbar)^3},$$

$$n_\pm = \frac{1}{2}\left(1 - \operatorname{th}\frac{p^2/2M \mp \beta H - \mu_\pm}{2T}\right). \qquad (3)$$

Here $\mathbf{p}$ are the momenta of the particles, $\beta = 0.08$ mK/kOe is the magnetic moment of the ^{3}He nucleus, and $\mu\pm$ are the chemical potentials of quasiparticles with a certain spin projection (if the spin polarization of the gas of ^{3}He quasiparticles is an equilibrium polarization and is determined by the value of the external magnetic field, then we would have $\mu_+ = \mu_-$).

We wish to stress that Eq. (2) is a universal function of the temperature and does not depend on a possible anisotropy of the spin dynamics or on the mechanism for the scattering of the ^{3}He quasiparticles (e.g., it does not depend on the role played by the scattering of impurity quasiparticles by phonons). Furthermore, function (2) can be calculated easily for any degree of degeneracy of the ^{3}He quasiparticle system, and it can be used to describe experimental data over a very broad temperature range, (1). The only conditions on the applicability of expression (2) are that the solution be dilute, $N_3a^3 \ll 1$, and condition (1).

Data on the $\Omega_{\rm int}\tau_\perp/D_\perp$ can be found from the experimental results of Ref. 12 for $\Omega_{\rm int}\tau_\perp$ and $D_\perp$ (Fig. 1) for $x = 3.7\cdot 10^{-4}$ and $H = 89$ kOe. The theoretical curves in this figure correspond to function (2) with $a = -0.5$ Å (curve 1) and $a = -1.5$ Å (curve 2). We see that curve 1 gives an excellent description of the experimental data over the entire temperature range. The data of Ref. 12 on $\Omega_{\rm int}\tau_\perp$ and $D_\perp$, taken separately, cannot give a satisfactory description over the entire temperature range, no matter how we choose a. At high temperatures, the difficulty comes from collisions with phonons and the violation of condition (1), while at low temperatures the difficulty comes from the anisotropy of the spin dynamics (Ref. 8, for example).

Because of the uncertainty in the data of Ref. 12 on the ratio $\Omega_{\rm int}\tau_\perp/D_\perp$, we can make only the following estimate of the scattering lines: $a = -0.5\div 0.7)$ Å. It is clear, however,

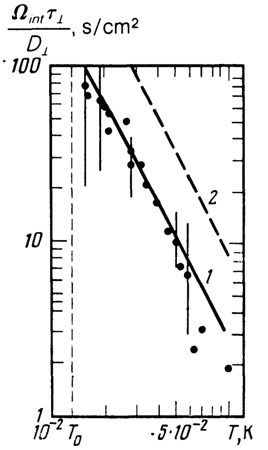

FIG. 1. Values of the parameters $\Omega_{\rm int}\tau_\perp/D_\perp$. Points: Experimental results of Ref. 12. Traces: Results of the present study. 1—At $a = -0.5$ Å; 2—at $a = -1.5$ Å.

that the value of $a = -1.5$ Å, taken from Refs. 5 and 13, is definitely too high. The value $a = -1.5$ Å was found in Ref. 13, primarily from the experimental values of D and $\varkappa$ for solutions which were degenerate and not very dilute, $x \gtrsim 10^{-2}$. The expression for the kinetic coefficients in Ref. 13, which were used to analyze the experimental data, are too high (by a factor of two for η and $\varkappa$ and by a factor of eight for D; cf. Ref. 14). The apparent reason why these values are too high is that Ref. 13 ignored the fact that particles with different spin projections participating in s scattering are not identical, and that the integration in the collision integral should be carried out over the entire phase space, rather than over only half of it. When we take this circumstance into account, we find that the difference between the experimental values used in Ref. 13 and the theoretical values of D and $\varkappa$ for the lowest-concentration solutions with $a = -0.7$ Å is the same as in Ref. 13 with $a = -1.5$ Å (the theoretical curve for D in Ref. 13 actually corresponds to the value a[Å] $= -1.5/8^{1/2} \sim -0.5$.

Table I shows data on a found from an analysis of experimental data on the kinetics of degenerate solutions. Since these results correspond to solutions with comparatively high ^{3}He concentrations, $x > 10^{-2}$, the data given for a are not very reliable.

Substituting the values $a = -(0.5\text{–}0.7)$ Å found from an analysis of the data of Ref. 12 into the expression for T_c (Refs. 5 and 19, for example) leads to the estimate

TABLE I.

Experimental data, $x = 0.013$	$-a$, Å	T_c, mK $x=0.03$
η [15]	0.75	0.03
$\varkappa$ [16]	0.83	0.06
D [17]	0.52	0.001
D [18]	0.54	0.0015
$\Omega_{int}\tau_\perp/D_\perp$ $x=3\cdot 10^{-4}$	0.5–0.7	0,0006–0,02

$T_c = 6\cdot 10^{-4} - 2\cdot 10^{-2}$ mK for a solution of concentration $x = 3\cdot 10^{-2}$. This figure does not look very optimistic (by wave comparison, with $a = -1.5$ Å and $x = 3\cdot 10^{-2}$ the value is $T_c = 1.4$ mK [Refs. 5 and 19; this estimate for T_c is clearly too high, according to the experimental data of Refs. 1–4 and 12]. Unfortunately, at present it is totally impossible to tell just how large the concentration x can be before the BCS theory will no longer give a satisfactory quantitative estimate of T_c or how T_c will behave at high concentrations (see, for example, Refs. 20 and 21 for some arguments regarding this point). If it turns out that detailed and accurate measurements yield $|a| \gtrsim 0.7$ Å, then attempts to observe a transition will be justified, while if $|a|$ is significantly smaller than 0.7 Å, such attempts would be pointless at present. Accordingly, it would be desirable, before the development of studies on cooling solutions to temperatures well below 1 mK, to first attempt to estimate a as accurately as possible in NMR experiments by one of the two methods which have been proposed. Another possibility is that effort should be focused not on an attempt to directly observe a transition to a superfluid state but to observe fluctuational effects above the transition (more on this below).

In summary, it can be concluded that the most promising and most accurate methods for determining the coupling constant are two methods for experimentally studying dilute $x \lesssim 10^{-3}$ solutions. The first method is measuring the spin diffusion coefficient in a dilute solution. The measurements would have to be carried out over a comparatively wide temperature range at ultralow temperatures, $T \lesssim 5$ mK, and in a weak magnetic field $\beta H \ll T_0$ (to eliminate an anisotropy of the spin fusion). The second method is to measure the ratio (2). This method does not require exceedingly low temperatures, but it does require comparatively strong magnetic fields, in order to achieve measurable values of $\Omega_{\rm int}\tau_\perp$.

To conclude this section we note that in experimental studies of the transition to the ^{3}He superfluid state in a solution at $T \lesssim 1$ mK there are two circumstances to be kept in mind. First, the experiment should be carried out in a comparatively weak magnetic field **H** (at a small degree of spin polarization P): $T_0 P/T, \beta H/T \ll 1$ (for a greater polarization, s pairing becomes impossible; see Ref. 5, for example). At $T_c < 1$ mK this limitation becomes important. The second circumstance stems from the fact that the BCS transition occurs only if the spin-lattice relaxation is fairly slow $\hbar/\tau^* < T_c$. In a ^{3}He–^{4}He solution, the time τ^* is determined either by an extremely weak nuclear magnetic dipole–dipole interaction, in which case there would be no limitations in practice, or—a more important consideration—by collisions of ^{3}He impurity quasiparticles with the wall with $\tau^* \sim L^2/wD$, where L is a length scale, and w is the spin accomodation coefficient at the surface. The latter circumstance turns out to be important at $T_c \lesssim 0.1$ mK, expecially for experiments in a bounded geometry, in which the solution is in a fine-pore medium for more effective cooling.

3. TRANSVERSE SPIN WAVES

The equations of spin dynamics for superfluid ^{3}He in a ^{3}He–^{4}He solution, as in the absence of superfluidity of ^{3}He (Ref. 5), break up into two separate systems of equations: equations of the transverse spin dynamics (the dynamics of the components of the magnetic moment perpendicular to the external magnetic field, **e**) and the equations of the longitudinal spin dynamics for the magnetic moment component M_z (the z axis is directed along **e**). In this section of the paper, we are concerned with the transverse spin dynamics.

There are several ways to describe phenomena close to equilibrium in superfluid Fermi systems. For long-wave processes, with $ka \ll 1$, the simplest is a semiclassical description of superfluid systems on the basis of a kinetic equation for the matrix distribution function (see, for example, the review by Serene and Rainer[22]). Since we are interested in the magnetic properties of a system of spin-1/2 particles, this kinetic equation will be a 4×4 matrix equation. It is more convenient for our purposes, however, to use a slightly more complicated description method, based on Green's functions. A description of this sort makes it a simple matter to generalize the results to the case of spatially inhomogeneous states of the solution (more on this below). This approach is actually equivalent to deriving a kinetic equation for such states.

We will describe the dynamics of a superfluid low-density gas of ^{3}He quasiparticles in solution by means of the system of Gor'kov equations for temperature-dependent Green's functions. In this case the coupling constant in the BCS theory, i.e., g, and the Fermi-liquid function are expressed in terms of the same quantity, a, i.e., the s-scattering length for ^{3}He quasiparticles[5]:

$$g = 4\pi |a| \hbar^2/M, \qquad \hat f = (2\pi a\hbar^2/M)(\hat I\hat I' - \hat{\sigma}\hat{\sigma}'),\quad a<0, \tag{4}$$

where $\hat I$ is the unit spin operator, and $\hat\sigma$ are the Pauli matrices. The f-functions in (4) do not depend on the momenta of the interacting quasiparticles because the s-scattering amplitude of slow particles does not depend on the momenta.

The Gor'kov equations with the Fermi-liquid interaction, (4), are derived in the usual way:

$$\left(-\frac{\partial}{\partial\tau} + \frac{\nabla^2}{2M} + \mu\right)\hat G + \frac{1}{2}\hbar\Omega_0\hat\sigma^z\hat G + ig\hat F(0)\hat F^+ - \hat\varepsilon_{int}\hat G = \delta(\mathbf{X}-\mathbf{X}')\hat I, \tag{5}$$

$$\left(-\frac{\partial}{\partial\tau} - \frac{\nabla^2}{2M} - \mu\right)\hat F^+ - \frac{1}{2}\hbar\Omega_0\hat\sigma^z\hat F^+ - ig\hat F^+(0)\hat G + \hat{\varepsilon}^{*}_{int}\hat F^+ = 0,$$

where $\hat G$ and $\hat F$ are the spin operators for the normal and anomalous temperature-dependent Green's functions, $\Omega_0 = 2\beta H/\hbar$ is the usual ordinary NMR frequency of ^{3}He, and the Fermi-liquid interaction is

$$\hat\varepsilon_{int}(\mathbf{X},\mathbf{X}') = \hat f\delta(\mathbf{X}-\mathbf{X}')\int \hat G(X',X'')\,d^4X''. \tag{6}$$

In this section of the paper we are interested in equations for the circular components of the Green's functions, $G^\pm = \mathrm{Tr}_\sigma(\hat\sigma^x \pm i\hat\sigma^y)\hat G$ and $F^\pm$:

$$\left(-\frac{\partial}{\partial\tau}+\frac{\nabla^2}{2M}+\tilde{\mu}-\frac{1}{2}\hbar\tilde{\Omega}\right)G^{+}+g\Xi F^{*+}+g\Xi^{+}F^{*1}-2\varepsilon_{int}G^{1}=0,$$

$$\left(-\frac{\partial}{\partial\tau}-\frac{\nabla^2}{2M}-\tilde{\mu}-\frac{1}{2}\hbar\tilde{\Omega}\right)F^{*+}+g\Xi G^{+}+g\Xi^{*+}G^{1}-2\varepsilon_{int}^{*+}F^{*1}=0, \quad (7)$$

where $\hat{\Xi}=\hat{F}^*(X,X)$ is the order parameter, $G^{1,2}=\mathrm{Tr}_\sigma(\hat{I}\pm\sigma^z)\hat{G}$ (there is an analogous expression for F^*), and $\tilde{\mu}$ and $\tilde{\Omega}$ are the chemical potential and NMR frequency, renormalized to incorporate the interaction and given by

$$\tilde{\mu}=\mu-\mathrm{Tr}_\sigma\hat{I}\hat{\varepsilon}_{int}=\frac{p_\pm^2}{2M}\mp\beta H\pm\frac{2\pi a\hbar^2}{M}(N_--N_+),$$

$$\tilde{\Omega}=\Omega_0-\Omega_{int},\quad \hbar\Omega_{int}\equiv\mathrm{Tr}_\sigma\hat{\sigma}^z\hat{\varepsilon}_{int}=-\frac{4\pi a\hbar^2}{M}(N_+-N_-) \quad (8)$$

Here $p_\pm=(6\pi^2N_\pm)^{1/3}$ are the Fermi momenta of particles with spin projections $\pm 1/2$ on the z axis, i.e., on **e**.

After linearization in terms of small perturbations and a Fourier transform, system (7), (8) reduces to

$$\mathcal{G}^{+}+\Delta^1F_-^{*1}G_+^2+\Delta^{*1}G_-^1F_+^{*2}-2\varepsilon_{int}G_-^1G_+^2-2\varepsilon_{int}^{*+}F_-^{*1}F_+^{*2}=0,$$

$$\mathcal{F}^{*+}+\Delta^{*1}G_-^1G_+^3+\Delta^1F_-^{*1}F_+^{*2}-2\varepsilon_{int}^{*+}F_-^{*1}G_+^3-2\varepsilon_{int}^{+}G_-^1F_+^{*2}=0, \quad (9)$$

where $\mathcal{G}$ and $\mathcal{F}$ are small deviations of the functions G and F from their equilibrium values, and $G_\pm^{(i)}$ and $F_\pm^{(i)}$ are the equilibrium Matsubara Green's functions of the superfluid Fermi gas of ^{3}He quasiparticles, given by

$$G_\pm^{(i)}=G^{(i)}(i\zeta_s\pm\omega/2,\mathbf{p}\pm\mathbf{k}/2),$$

$$F_\pm^{(i)}=F^{(i)}(i\zeta_s\pm\omega/2,\mathbf{p}\pm\mathbf{k}/2),$$

$$G^{1,2}(\zeta_s,\mathbf{p})=\frac{i\zeta_s+(p^2-p_\mp^2)/2M}{[i\zeta_s+(p^2-p_\mp^2)/2M][i\zeta_s-(p^2-p_\pm^2)/2M]-\Delta^2},$$

$$G^{3,4}=-G^{*1,2}, \quad (10)$$

$$F^{*1,2}(\zeta_s,\mathbf{p})=-\frac{\Delta}{[i\zeta_s+(p^2-p_\mp^2)/2M][i\zeta_s-(p^2-p_\pm^2)/2M]-\Delta^2},$$

$$F^{*1,2}=F^{2,1}.$$

Here $\zeta_s=\pi T(2s+1)$ is the Matsubara frequency, and at $\Delta^1=g\Xi^+$ is the deviation of the energy gap from its equilibrium value Δ. Since both Δ^1 and ε_{int}, (on the one hand) and Δ^{*1} and ε_{int}^* (on the other) appear in Eqs. (9), these equations must be supplemented with conjugate equations for $\mathcal{G}^*$ and $\mathcal{F}$.

Since the f-function in (4) does not depend on the momenta, it is an elementary matter to solve (9), a system of four integral equations. As a result, the spectrum of transverse spin waves, $\omega(k)$, can be described implicitly by the equation

$$CC^*EE^*+D(CC^*+EE^*)+(AA^*-BB^*)^2$$
$$+\{[-CEA^{*2}-BB^*CE^*+DA^*(CB^*+CB+EB^*+EB)]+\text{c.c.}\}=0, \quad (11)$$

where

$$A=T\sum_{s=-\infty}^{\infty}\int d\Gamma\,G_-^1F_+^{*2},\quad B=T\sum_{s=-\infty}^{\infty}\int d\Gamma\,F_-^{*1}G_+^2,$$

$$C=T\sum_{s=-\infty}^{\infty}\int d\Gamma\,G_-^1G_+^2-\frac{1}{g},$$

$$E=T\sum_{s=-\infty}^{\infty}\int d\Gamma\,G_-^1G_+^3+\frac{2}{g},$$

$$D=T\sum_{s=-\infty}^{\infty}\int d\Gamma\,F_-^{*1}F_+^{*2},\quad d\Gamma=d^3\mathbf{p}/(2\pi\hbar)^3. \quad (12)$$

For arbitrary Δ/T the spin-wave spectrum $\omega(k)$ found by solving Eqs. (11) is rather complicated. Near the transition, at $\Delta/T\ll 1$, Eq. (11) simplifies significantly. We are interested in the spin-wave spectrum near the NMR frequency, $\omega-\Omega_0\ll\Omega_{int}$. At $\Delta=0$, the corresponding spectrum is determined by the equation $C=0$. At $0<\Delta/T\ll 1$, Eq. (11) reduces, to within terms $(\Delta/T)^2$, to the following equation near the NMR frequency:

$$CE=2A^2. \quad (13)$$

At $\Delta/T\ll 1$, it is an elementary matter to evaluate the integrals in (10) and (12). As expected, the appearance of an energy gap Δ does not shift the frequency of the uniform NMR: $\omega(k=0)=\Omega_0\equiv 2\beta H/\hbar$. The appearance of a gap in the long-wave region leads to corrections of the type Δ^2k^2 to the spectrum. In principal, the terms Δ^2k^2 in Eq. (13) have several sources and differ by factors on the order of $|a|N_3^{1/3}$. Since our equations are written in the first approximation in $|a|N_3^{1/3}$, we should retain in the spectrum only the leading term in this parameter.

We are interested in the long-wave region, $kv_0\ll\Omega_{int}$, in which the spectrum is quadratic: $\omega-\Omega_0\propto k^2$. At these frequencies, the damping of the oscillations is determined by the parameter $\Omega_{int}\tau$ and does not depend on $\omega\tau$ (Ref. 8, for example). The integrals of the Matsubara Green's functions which appear in (13) can always be evaluated in this case, as in ordinary low-frequency hydrodynamic problems, and results can be found by expanding in kv_0/Ω_{int}, $(\omega-\Omega_0)/\Omega_{int}$. As a result, spin-wave spectrum (13) takes the following form, at our accuracy level:

$$\omega-\Omega_0=\frac{k^2v_0^2}{3\Omega_{int}}\left\{1+\frac{(36\pi)^{1/3}}{8\pi^2}\left[\frac{\Delta}{T_c(H)}\right]^2\times\sum_{s=1}^{\infty}\frac{1}{(2s-1)^3}\frac{1}{(1+h_s^2)^2}\right\}$$

$$h_s=\frac{\beta H}{T_c(H)}\frac{1}{(2s-1)\pi}. \quad (14)$$

In the case $\Delta=0$, spectrum (14) is evidently the same as the spin-wave spectrum in a degenerate, low-density Fermi gas[5] in a weak magnetic field, $(N_+-N_-)/N_3=3\beta H/T_0\ll 1$. The magnetic field has been assumed to be weak on the scale of T_0, but not of T_c, in (14).

We should stress that the correction $(\Delta/T)^2$ in (14) results from a change in the equilibrium degree of spin polarization of the solution in a static magnetic field as a result of the appearance of superfluidity. The degree of polarization of the solution changes only as a result of extremely slow processes of a magnetic dipole-dipole interaction or as a result of collisions of ^{3}He quasiparticles with the walls. Consequently, the superfluid transition first occurs at a constant magnetic moment of the solution, and, at our level of accuracy, there are no corrections on the order of Δ^2 in the spectrum. Such corrections arise only after the equilibrium magnetization is established (the typical time τ^* may prove very long if the magnetic coupling of ^{3}He quasiparticles with the walls is not good).

In magnetic fields which are not too weak, and at $T \lesssim T_c$, the relation $\Omega_{\rm int}\tau \gg 1$ usually holds. The damping of spin waves (14) in this case is $\omega'' = 1/\Omega_{\rm int}\tau$, and the real part of the spectrum also has small corrections of order $1/\Omega_{\rm int}\tau$ for the Leggett-Rice effect.[10]

In the absence of superfluidity, $\Delta = 0$, the temperature dependence of the frequency shift and of the attenuation of the NMR results exclusively from the temperature dependence $\tau(T)$. Under the conditions $\Omega_{\rm int}\tau \gg 1$ and $\Delta = 0$, the frequency shift does not depend on the temperature. As the temperature is lowered further, a temperature dependence of the frequency shift due to $\Delta(T) \neq 0$ reappears near the superfluid transition. Near the transition this temperature dependence is determined by the second term in (14): $\Delta^2 \propto T_c(T_c - T)$. However, a temperature dependence of this sort can also be observed slightly above the transition, because of superfluid fluctuations above the transition, for which we have the following:

$$\Delta^2 \approx \frac{16\pi^2 T}{7\zeta(3)}(T-T_c)\left\{\left[1+\frac{7\zeta(3)\hbar^3 kT}{2MVp_F(T-T_c)^2}\right]^{1/2}-1\right\}.$$

The apperance of a specific temperature dependence for the NMR frequency shift would therefore be one of the clearest indictations of an approach to a superfluid transition. It is possible that the temperature dependence of the frequency shift which was noted in the experiments of Ref. 23, at the extremely low temperatures attained there, is evidence of an approach to a transition.

These results can be generalized easily to the case of spatially inhomogeneous phases of ^{3}He in solution at $\beta H \gg 1.06 T_c$ $(H=0)$ (Ref. 5). In this case the order parameter is

$$\Delta(\mathbf{r}) = \Sigma\Delta_n \exp(i\mathbf{Q}_n\mathbf{r}/\hbar), \qquad |\mathbf{Q}_n| = Q(H),$$

and the second term in braces in (14) should be replaced by the following expression at $k \ll Q/\hbar$:

$$\frac{(36\pi)^{1/3}}{8}\frac{F_1(H)}{(\beta H)^2}\sum_n |\Delta_n|^2, \qquad (15)$$

where the function

$$F_1 = \frac{2\beta H}{Qv_0}\sum_{s=1}^{\infty}\frac{1}{2s-1}\{[1+(q_s-h_s)^2]^{-1}-[1+(q_s+h_s)^2]^{-1}\},$$
$$q_s = Qv_0/2\pi(2s-1)T_c(Q,H),$$

is plotted in Ref. 5. It is a simple matter to use (15) to reconstruct the macroscopic equation of motion of the magnetic moment of the solution in inhomogeneous phases at $k \ll Q/\hbar$.

4. LONGITUDINAL SPIN-SOUND WAVES

In the absence of superfluidity of ^{3}He in a solution in a weak magnetic field, coupled spin-sound waves may propagate with a linear dispersion law, $\omega = ck$. These waves would be determined by simultaneous solution of the coupled high-frequency kinetic equation for the scalar distribution function of the ^{3}He quasiparticles and the longitudinal component (parallel to e) of the distribution of the magnetic moment. When ^{3}He superfluidity is present, the corresponding equations for the Green's function reduce to the following system of four equations, according to (5):

$$\mathscr{G}^{1,2}+\Delta^{1,2}F_-^{*1,2}G_+^{1,2}+\Delta^{*1,2}G_-^{1,2}F_+^{*1,2}-2\varepsilon_{int}^{1,2}G_-^{1,2}G_+^{1,2}$$
$$+2\varepsilon_{int}^{*2,1}F_-^{*1,2}F_+^{*1,2}=0,$$
$$\mathscr{F}^{*1,2}+\Delta^{*1,2}G_+^{4,3}G_-^{1,2}+\Delta^{1,2}F_-^{*1,2}F_+^{1,2}+2\varepsilon_{int}^{*2,1}F_-^{*1,2}G_+^{4,3}$$
$$-2\varepsilon_{int}^{1,2}G_-^{1,2}F_+^{*1,2}=0, \qquad (16)$$

where $\mathscr{G}$, $\mathscr{F}$ and $\Delta^{1,2} = g\Xi^{1,2}$ are the deviations of the Green's functions and of the gap from their equilibrium values, determined from $X^{1,2} = \mathrm{Tr}(\hat{I} \pm \hat{\sigma}^z)\tilde{X}$.

System (16) should be supplemented with four adjoint equations. However, the system of eight equations which arises is degenerate since the quantities $\Delta^{1,2}$ appear only in the combinations $\Delta^1 + \Delta^{*1}$ and $\Delta^2 + \Delta^{*2}$. For this reason, the spin-wave spectrum is determined by sixth-order determinant. It should be kept in mind that the waves in which we are interested are high-frequency waves, and in calculating integrals of the Green's functions we should draw a distinction between retarded and advanced functions (no significant complication results).

In the absence of superfluidity of ^{3}He and in the absence of a magnetic field, in a solution with $a<0$, high-frequency spin waves may propagate at a velocity c which is exponentially close to the Fermi velocity v_0 at low ^{3}He concentration, $p_0|a|/\hbar \ll 1$:

$$\ln(1/\alpha_0) = \hbar\pi/p_0|a| \gg 1, \quad \alpha_0 = (c-v_0)/v_0 > 0, \quad c = \omega/k. \qquad (17)$$

Correspondingly, near the superfluid transition, $\Delta/T \ll 1$, and in weak magnetic fields, we should restrict the calculation of the coefficients in the dispersion relation exclusively to the lowest-order terms according to the large logarithms $\ln[v_\pm/(c-v_\pm)]$ [to incorporate the terms of higher order according to these logarithms, which correspond to the coefficients of the exponential functions in (17), would be to go beyond the accuracy of this treatment for the f-functions in (4)].

In the absence of a spin polarization, $p_+ = p_-$, the dispersion relation reduces to

$$L = -\frac{v_0 M^2}{4\pi^2\hbar^3}\left\{\ln\alpha + \frac{\Delta}{4T}\left[\ln\alpha + \frac{\pi}{(2\alpha)^{1/2}}\right]\right\} = \frac{1}{g}. \qquad (18)$$

This expression holds only near the transition, at $\Delta/T \ll \alpha^{1/2} \ll 1$. The result (18) is not very interesting, however, since we known[6,7] that the appearance of superfluidity for a slightly nonideal system would lead not only to a pronounced change in the spin-wave spectrum ($\alpha^{-1/2} \gg |\ln \alpha|$) but also to a rapid increase in the collisional damping of waves, determined by the parameter $\exp(2T_0\alpha/T)$. This damping would not be small even at $T \gg T_c$. Collisionless damping prevent the observation of high-frequency spin waves over essentially the entire temperature range, both above and below T_c.

Imposing an external magnetic field changes the situation fundamentally. First, the polarization of the solution reduces the temperature of the superfluid transition, thereby complicating an experimental study below the transition. Second, it may, however, cause a pronounced decrease in the collisionless damping near the transition. At $\hbar\Omega_0 \gg \Delta$, $T_c(H)$, for example, the dispersion relation becomes

$$L_+L_- = \frac{1}{g^2}, \quad L_\pm = \frac{cM^2}{4\pi^2\hbar^3}\left(1+\frac{\Delta^2}{2\hbar^2\Omega_0^2}\right)\ln\left(\alpha_\pm + \frac{\Delta^2}{2\hbar^2\Omega_0^2}\right),$$
$$\alpha_\pm = (c - v_\pm)/v_\pm. \tag{19}$$

Expression (19) shows that as the magnetic field is increased the propagation velocity of the spin waves, c, rapidly approaches v_+; the correction for Δ is most important in the argument of the logarithm. In strong magnetic fields (or at substantial values of $\Delta/\hbar\Omega_0$) we find $\alpha_+ \to 0$, $\alpha_- \to (v_+ - v_-)/v_-$, and

$$\alpha_+ \equiv \frac{c - v_+}{v_+} = -\frac{\Delta^2}{2\hbar^2\Omega_0^2} + \exp\left\{\frac{\pi^2 a^{-2}(6\pi^2 N_+)^{-2/3}(1+\Delta^2/2\hbar^2\Omega_0^2)^2}{\ln[\Delta^2/2\hbar^2\Omega_0^2 + (v_+ - v_-)/v_-]}\right\}.$$

These results determine the maximum values of Δ/Ω_0 at which the waves can still propagate ($\alpha_+ > 0$). Collisionless damping does not prevent an observation of spin waves (19) in magnetic fields ($H > H_0$) in which the temperature of the superfluid transition, $T_c(H)$, is quite low:

$$\exp\{T_0\alpha_+(H_0)/T_c(H_0)\} \gg 1. \tag{20}$$

The function $T_c(H)/T_c(H=0)$ is plotted in the review by Bashkin and Meyerovich,[5] among other places ($T_c(H=0) \sim T_0 \exp[-\pi/2|a|(3\pi^2 N_3)^{1/3}]$). In principle, since s pairing would be totally impossible in fields $H > H_s$, $\beta H_s = 1.33T_c(H=0)$, theoretically there would always be a region of parameter values in which weakly damped longitudinal spin waves could propagate. Unfortunately, the relation $T_c(H_0) \ll T_c(H=0)$ holds, and the observation of weakly damped ($H > H_0$) longitudinal spin waves in a solution seems unlikely at present.

However, expression (19) determines the propagation velocity of spin waves in an ordinary slightly nonideal Fermi gas. In a ^{3}He–^{4}He solution in a magnetic field, the propagation of longitudinal spin waves is known[5] to be accompanied by oscillations of the Bose background. To take this circumstance into account, we should supplement system (16) with a continuity equation and an equation for the superfluid motion. The corresponding linearized equations are (cf. Ref. 5)

$$-\omega\left[m_4\delta N_4 + \frac{1}{2}m_3\int\frac{d^3p}{(2\pi\hbar)^3}(\mathcal{G}^1+\mathcal{G}^2)\right] + \mathbf{k}\left[\rho_s\mathbf{v}_s + \frac{1}{2}\int\frac{\mathbf{p}\,d^3p}{(2\pi\hbar)^3}(\mathcal{G}^1+\mathcal{G}^2)\right] = 0,$$

$$-\omega\mathbf{v}_s - \frac{\mathbf{k}^2}{m_4}\left[\frac{m_4^2}{\rho_s}s_4^2\delta N_4 + \frac{\partial\varepsilon_0}{\partial N_4}\int\frac{d^3p}{(2\pi\hbar)^3}(\mathcal{G}^1+\mathcal{G}^2)\right] = 0,$$

where s_4 is the sould velocity in pure ^{4}He, $\mathbf{v}_s$ is the superfluid velocity of ^{4}He, δN_4 is the oscillatory increment in the ^{4}He density, $\rho_s = m_4N_4 + (m_4 - M)N_3$, N_4 is the number of ^{4}He atoms per unit volume, m_4 is the mass of the ^{4}He atom, and ε_0 is the lowering of the energy of HeII due to the addition of one ^{3}He atom, $(N_4/m_4s_4^2)\times(\partial\varepsilon_0/\partial N_4) \sim 1.28$. In this case we should add to the Hamiltonian of the ^{3}He quasiparticles a term describing the change in the energy of a ^{3}He quasiparticle due to a change in the ^{4}He density, $(\partial\varepsilon_0/\partial N_4)\delta N_4$, and due to the appearance of superfluid motion of ^{4}He, $\mathbf{p}\mathbf{v}_s(1 - m_3/M)$. Correspondingly, we would add terms

$$-\frac{\partial\varepsilon_0}{\partial N_4}\delta N_4(G_-^{1,2}G_+^{1,2} + F_-^{*1,2}F_+^{*1,2})$$

$$+\left(1 - \frac{m_3}{M}\right)\mathbf{p}\mathbf{v}_s\left(G_+^{1,2}\frac{\partial G_-^{1,2}}{\partial\mathbf{k}\mathbf{v}} + F_+^{*1,2}\frac{\partial F_-^{*1,2}}{\partial\mathbf{k}\mathbf{v}}\right)$$

to the first of Eqs. (16) and add analogous terms to the second two equations (16). A system of equations of this sort, describing the propagation of coupled spin-sound waves, turns out to be rather complicated. It can be shown that near the transition the propagation velocity of the spin-sound waves is given by the following equation under the conditions $\hbar\Omega_0 \gg \Delta$, $T_c(H)$ [cf. (19)]:

$$1 - \frac{1}{2}\frac{2\pi^2\hbar^3}{M^2}\Phi_0(L_+ + L_-) + \frac{gv_0}{M^2}\left(\frac{2\pi^2\hbar^3}{M^2}\Phi_0 - \frac{gv_0}{M^2}\right)L_+L_- = 0, \tag{21}$$

$$\Phi_0 \equiv -3m_4(s_0^2N_3/p_0v_0N_4)(N_4/m_4s_0^2)^2(\partial\Delta/\partial N_4)^2 = -\varphi_0 x^{1/3},$$

where x is the ^{3}He concentration, and functions $L_\pm$ are determined by expression (19) (at the saturation vapor pressure, we would have $\varphi_0 = 24.2$). At $\Delta = 0$, Eq. (21) has a solution[5] $c > v_+$ only in magnetic fields $H < H_c$:

$$H_c = T_0\exp\left[-\frac{\varphi_0/x^{1/3}}{Z(\varphi_0+Z)}\right], \quad Z = 2|a|\left(\frac{3N_4}{\pi}\right)^{1/3}. \tag{22}$$

This result is equivalent to the following numerical relation at the saturation vapor pressure

$$H_c\,[\text{Oe}] \sim 10^7x^{2/3}\exp[-78.4/|a|(43.6+|a|)x^{1/3}],$$

where a is expressed in angstroms. It is not difficult to see that at $x \ll 1$ the field H_c given by (22) is far higher than not only the field H_0 in (20) but also the field $\beta H_s = 1.33T_c(H=0)$,

$$H_s\,[\text{Oe}] \sim 10^7x^{2/3}\exp[-1.8/x^{1/3}|a|], \tag{23}$$

above which s paring is totally impossible. We thus conclude that weakly damped spin-sound waves in a solution can be observed in a finite field interval $H_0 > H > H_c$. At $H_0 > H > H_s$, these waves can be observed near the transition and in the superfluid phase of ^{3}He. For $H > H_s$, we should not retain terms with Δ^2 in L_+ in (21):

$$1-6.05x^{1/3}(\ln\alpha_+ + \ln\alpha_-) - \frac{|a|}{7.6}\left(24.2+\frac{|a|}{1.8}x^{2/3}\ln\alpha_+\ln\alpha_-\right)=0.$$

For $H < H_s$, we can incorporate in this equation for α corrections of order Δ^2, which correspond to the replacements $x^{1/3}\to x^{1/3}(1+\Delta^2/2\hbar^2\Omega_0^2)$, $\alpha_\pm \to \alpha_\pm + \Delta^2/2\hbar^2\Omega_0^2$. The dependence $T_c(H)$ is of such a nature that the condition $2\beta H \gg T_c(H)$—a necessary condition for the expansion in Δ to be carried out in terms of the parameter $\Delta/\hbar\Omega_0$—corresponds to the case in which the field H should not be much weaker than H_s.

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