

## TRANSVERSE SPIN DYNAMICS IN SPIN-POLARIZED FERMI LIQUIDS

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A microscopic framework for a generalized Landau theory is established in the form of two coupled equations in "partial transverse densities". The 4-component effective interaction is related to the irreducible vertex, and not to the full vertex. The results explain the zero-temperature transverse relaxation and attenuation of spin waves. The spectrum of spin waves is expressed via harmonics of the interaction operator and its derivatives at arbitrary polarization. Our exact equations differ from previous semi-phenomenological ones, and reproduce all proper limiting cases like spin-polarized quantum gases or the Silin-Leggett low field equations.

### 1. INTRODUCTION

The simplest multilevel Fermi liquids are spin-polarized or two-level systems. The transverse dynamics of multilevel systems describes the off-diagonal components of single-particle operators. In spin-polarized systems, it covers the dynamics of the transverse components of the magnetic moment. The transverse dynamics in a two-level system is the same as in a polarized system with (pseudo-)spins  $1/2$ .

The longitudinal dynamics of Fermi liquids for pure spin states, is quite clear, and obeys the standard Landau theory. It is not so for transverse dynamics [1] because of an unclear concept of attenuating off-diagonal quasi-particles and a non-locality. We made a microscopic attempt to modify the Landau theory in order to make it work for transverse phenomena. As

a result, all the equations undergo major changes. The theory is highly non-local with a large zero-temperature attenuation, and cannot be easily reproduced phenomenologically. The equations have a Landau-like appearance, but the underlying physics is different. Sec.2 contains the main equations. In Sec.3 we dwell on the meaning of these equations and their consequences. The details will be published elsewhere.

## 2. MAIN RESULTS

We reduced the Dyson equation in the off-diagonal component of the single-particle Green's function  $G_{\uparrow\downarrow}$  to an integral equation with an irreducible two-particle vertex in the kernel. The aim is to reduce the equation in  $G_{\uparrow\downarrow}(K;P)$  (the 4-vectors  $P = (\varepsilon, \mathbf{p})$ ,  $K = (\omega, \mathbf{k})$  describe the 4-momentum and temporal and spatial non-localities) to an equation in an off-diagonal density  $\delta n_{\uparrow\downarrow}(\mathbf{k};\mathbf{p})$ . This must be done by excluding the energy component of the 4-vector  $P$  uniformly throughout the equation. This is impossible because of unavoidable temporal non-localities. The only option is to reduce the equation in  $G_{\uparrow\downarrow}$  to a set of two coupled equations in "partial transverse densities"  $\delta n_{\uparrow}(\mathbf{p}) \equiv \delta n_{\uparrow\downarrow}^{(\uparrow)}(\mathbf{p})$  and  $\delta n_{\downarrow}(\mathbf{p}) \equiv \delta n_{\uparrow\downarrow}^{(\downarrow)}(\mathbf{p})$  which correspond to the same function  $G_{\uparrow\downarrow}$  but are centered around different frequencies:

$$\begin{aligned} \pm (\delta\omega - \frac{m'_{\uparrow} + m_{\downarrow}}{2m'_{\uparrow}m_{\downarrow}} \mathbf{p}\mathbf{k}) \delta n_{\uparrow\downarrow}(\mathbf{p}) &= \frac{1}{2} \sum_{\uparrow\downarrow} \int F_{\uparrow\uparrow}(K; \mathbf{p}, \mathbf{p}') \delta n_{\uparrow}(\mathbf{p}') \times \\ [\theta_{\downarrow}(\mathbf{k}; \mathbf{p}') - \theta_{\uparrow}(\mathbf{k}; \mathbf{p}')] \frac{d^3 p'}{(2\pi)^3} - \frac{1}{2} \delta n_{\uparrow\downarrow}(\mathbf{p}) \sum_{\uparrow\downarrow} \int F_{\uparrow\uparrow}(K_0; \mathbf{p}, \mathbf{p}') \times & (1) \\ [\theta_{\downarrow}(\mathbf{k}=0; \mathbf{p}') - \theta_{\uparrow}(\mathbf{k}=0; \mathbf{p}')] \frac{d^3 p'}{(2\pi)^3}, & \end{aligned}$$

(the double-arrow indices  $\uparrow, \downarrow$  take two values  $\uparrow$  and  $\downarrow$ ). Two Eqs.(1) substitute the standard Landau equation for transverse phenomena. The changes do not end with this doubling of equations with 4 generalized transverse Landau functions  $F_{\uparrow\uparrow}$  and 4 relaxation times  $\tau_{\uparrow\uparrow}$ . We will describe other differences from the naive models by explaining the notations in Eqs.(1).

The interaction function is not related to the full vertex as for of non-polarized systems. Instead, the non-local 4-component kernel  $F_{\uparrow\uparrow}(K; \mathbf{p}, \mathbf{p}')$  is given by the mixed component of the irreducible vertex  $\Gamma_{\uparrow\downarrow, \downarrow\uparrow}(K; P, P')$ :

$$\begin{aligned} F_{\uparrow\downarrow}(K; \mathbf{p}, \mathbf{p}') &= Z_{\downarrow}(\mathbf{p}' \pm \frac{\mathbf{k}}{2}) \mathbb{G}(K; P_{\uparrow}, P_{\downarrow}), \\ \mathbb{G}(K; P, P') &= \Gamma_{\uparrow\downarrow, \downarrow\uparrow}(K; P, P') + \\ \int \Gamma_{\uparrow\downarrow, \downarrow\uparrow}(K; P, Q) \phi_r(Q+K/2, Q-K/2) \mathbb{G}(K; Q, P') \frac{d^4 Q}{(2\pi)^4} & (2) \end{aligned}$$

(+ corresponds to  $Z_{\downarrow}$ ). Only at low polarizations the solution of Eq.(2) is equal to the limit  $\Gamma^0$  of the full vertex  $\Gamma$ .

The functions  $Z_{\uparrow}(\mathbf{p})$ ,  $\phi_r$ , and  $\phi_s$  are the residue, regular and singular parts of the Green's functions  $G_{\uparrow\uparrow}$  and  $G_{\downarrow\downarrow}$  for the pure spin states at the Fermi surfaces,

$$\begin{aligned}
 G_{\downarrow\downarrow}(P+\frac{K}{2}) - G_{\uparrow\uparrow}(P-\frac{K}{2}) &= i\phi_s(P+\frac{K}{2}, P-\frac{K}{2}) + \\
 &+ i\phi_r(P+\frac{K}{2}, P-\frac{K}{2}) [\delta\omega - \frac{\mathbf{p}\mathbf{k}}{m} - \Sigma_{\downarrow\downarrow}(P+\frac{K}{2}) + \Sigma_{\uparrow\uparrow}(P-\frac{K}{2})], \\
 \phi_s(P+\frac{K}{2}, P-\frac{K}{2}) &= \pi [\theta(\varepsilon_{\downarrow}(P+\frac{\mathbf{k}}{2}) - \mu) - \theta(\varepsilon_{\uparrow}(P-\frac{\mathbf{k}}{2}) - \mu)] \times \\
 &[Z_{\downarrow}(P+\frac{\mathbf{k}}{2}) \delta(p_0 + \frac{\delta\omega}{2} + \beta H - \varepsilon_{\downarrow}(P+\frac{\mathbf{k}}{2}) + \mu) + \\
 &Z_{\uparrow}(P-\frac{\mathbf{k}}{2}) \delta(p_0 - \frac{\delta\omega}{2} - \beta H - \varepsilon_{\uparrow}(P-\frac{\mathbf{k}}{2}) + \mu)], \\
 1 - \frac{1}{Z_{\uparrow}(\mathbf{p})} &= \frac{\partial \Sigma_{\uparrow\uparrow}(p_0 = \varepsilon_{\uparrow}(\mathbf{p}), \mathbf{p})}{\partial p_0}
 \end{aligned} \tag{3}$$

where  $\Sigma_{\uparrow}(\mathbf{p})$  are the mass operators for the pure (spin) states.

The masses in Eqs.(1) are given by the momenta derivatives of the energies  $\varepsilon_{\uparrow}$  and some pseudo-energies  $\varepsilon'_{\uparrow}$ :

$$\begin{aligned}
 \varepsilon_{\uparrow}(\mathbf{p}\mp\frac{\mathbf{k}}{2}) &= \frac{p^2}{2m} + \Sigma_{\uparrow\uparrow}(\varepsilon_{\uparrow}(\mathbf{p}\mp\frac{\mathbf{k}}{2}) - \mu, \mathbf{p}\mp\frac{\mathbf{k}}{2}) \mp \beta H, \\
 \varepsilon'_{\uparrow}(\mathbf{p}\mp\frac{\mathbf{k}}{2}) &= \frac{p^2}{2m} + \Sigma_{\uparrow\uparrow}(\varepsilon_{\uparrow}(\mathbf{p}\pm\frac{\mathbf{k}}{2}) - \mu \mp \delta\omega \\
 &\pm 2(\beta_1(\mathbf{p}; \mathbf{k}, H) - \beta) H, \mathbf{p}\mp\frac{\mathbf{k}}{2}) \mp \beta H, \\
 \frac{\mathbf{p}}{m_{\uparrow}} &= \frac{\partial \varepsilon_{\downarrow}}{\partial \mathbf{p}}, \quad \frac{\mathbf{p}}{m'_{\uparrow}} = \frac{\partial \varepsilon'_{\downarrow}}{\partial \mathbf{p}},
 \end{aligned} \tag{4}$$

where in  $\mp$  the upper sign corresponds to  $\uparrow$ , and the function

$$\beta_1(\mathbf{p}; \mathbf{k}, H) = \frac{1}{2H} [\varepsilon_{\uparrow}(P-\frac{\mathbf{k}}{2}) - \varepsilon_{\downarrow}(P+\frac{\mathbf{k}}{2})] \tag{5}$$

characterizes the magnetic susceptibility of the system. According to Eqs.(4), the pseudo-energies  $\varepsilon'_{\uparrow}$  are close to the energies  $\varepsilon_{\uparrow}$  only if the polarization/field is low or the density and/or the interaction are small ( $\beta_1$  close to  $\beta$ ). In both cases two Eqs.(1) collapse into a standard one.

The most important notation is the definition of the partial "densities"  $\delta n_{\uparrow\downarrow}(\mathbf{p})$  through the Green's function:

$$\begin{aligned}
 \delta n_{\uparrow}(\mathbf{p}) &= g(P_{\uparrow}), \quad \delta n_{\downarrow}(\mathbf{p}) = g(P_{\downarrow}), \\
 \delta G_{\uparrow\downarrow}(P; t, \mathbf{x}) &= g(P) [G_{\downarrow\downarrow}(P+\frac{K}{2}) - G_{\uparrow\uparrow}(P-\frac{K}{2})] \exp(i\mathbf{k}\mathbf{x} - i\omega t)
 \end{aligned} \tag{6}$$

### 3. DISCUSSION AND SUMMARY

Eqs. (1) only mimic the Landau equation. Two partial densities  $\delta n_{\downarrow}(\mathbf{p}) \equiv \delta n_{\uparrow}^{(\downarrow)}(\mathbf{p})$  are complex and do not constitute long-lived quasi-particles, but are partial contributions to the off-diagonal component of the density matrix,  $\delta n_{\uparrow\downarrow}(\mathbf{p}) = \text{Tr}_\sigma \sigma^+ \delta \hat{n}_\sigma(\mathbf{p})$ .

Generically, these densities are the distributions for transverse magnetic moments originating from slightly tilted spins of spin-up and spin-down particles. Tilted spin-ups and spin-downs precess around their effective fields. The effective fields are the sums of the external field  $H$ , and the "molecular" field caused by a re-distribution of surrounding particles (the quotation marks emphasize that this field is complex and just looks like a molecular field). The molecular fields are proportional to the gradients of magnetization and vanish at  $\mathbf{k} = 0$ . Therefore, in a homogeneous setting  $\mathbf{k} = 0$ , the precession frequencies for both spin-ups and spin-downs are equal to the Larmor frequency  $2\beta H$ . This can be seen from Eqs. (1): at  $\mathbf{k} = 0$  the solution is  $\delta\omega = 0$ ,  $\delta n_{\uparrow}(\mathbf{p}) = \delta n_{\downarrow}(\mathbf{p}) = \text{const.}$

If  $\mathbf{k} \neq 0$ , the molecular fields are finite. The spin-ups and spin-downs are dressed differently making their molecular fields and precession frequencies, are different. This leads to a strong dephasing and to an inhomogeneous broadening of NMR spectra causing the zero-temperature attenuation.

We solved Eqs. (1) and expressed the spectrum of spin waves via the harmonics of the interaction functions  $F_{\uparrow,\downarrow}$  and their derivatives at arbitrary degrees of spin polarization. However, the result is too cumbersome to be given here.

The exact microscopic equations (1) describe the spin dynamics in all polarized Fermi liquids. These equations give a basis for calculations for different types of systems, and indicate how to introduce a proper macroscopic theory. An additional analysis for limiting cases of low polarizations or densities, and long-range interaction is given in Ref. [2].

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### REFERENCES

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